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# NOTES ON FRANKLIN'S ARMONICA AND THE MUSIC MOZART WROTE FOR IT

BY

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During 1956 the Western World has paid tribute to two of its historic figures. Franklin has had his 250th anniversary and Mozart his 200th. But these great men have more in common than an anniversary—Mozart the composer and Franklin, in his least known role, the musician.

Because Franklin was musical from his early years—playing the harp, violin and guitar—he had a firm foundation for a lifetime of playing and appreciating. Even more, he had mastery of music's fundamentals. That explains why he could tinker with a musical novelty or parlor trick and, using its components, invent an instrument.

Because the instrument played its way through Europe's cities to its musical capital, Vienna, it fired the fancy of history's most astonishing composer, and Mozart wrote music for Franklin's Armonica.

Thus the names of Franklin and Mozart—both musicians and democrats—are linked in the democracy of music.

## THE MUSICAL GLASSES WERE THE INSPIRATION

Franklin's second trip to England was his first on official business. The year 1757 found him, at age 51, retired nine years from business, celebrated for his experiments in electricity, and Britain-bound to air a colonial grievance as Agent of Pennsylvania.

In London, Franklin found the Musical Glasses in vogue. They were the drinking glasses of various sizes, arranged in a row on a board and filled with varying amounts of water so each had a different pitch or tone. A wet finger passed around the brims produced musical strains of delicate beauty. Performers also drew sounds by tapping the glasses with sticks. The Musical Glasses had many functions—a parlor game,

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(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.

a musical hobby or accomplishment, a theatre act and on occasion a concert novelty. About ten years before (1746) Gluck, the promising German composer of operas, had given a concert featuring the Musical Glasses with orchestral accompaniment. This was at the Little Theatre at the Haymarket, in London. For his audience says *Grove's Dictionary of Music and Musicians* (*1940*), he played

> . . . a concerto on 26 drinking glasses tuned with spring water, accompanied with the whole band, being a new instrument of his own invention; upon which he performs whatever may be done on a violin or harpsichord.

The reference to "his own invention" undoubtedly means, not literally an invention, but an improvement or application. Centuries before Gluck, the glasses were known in the Orient and the Middle East. They "spread from the Orient westward through the hardy travelers," says A. Hyatt King,[2] British scholar whose research has traced the glasses to the Thirteenth Century.

A Chinese encyclopedia compiled in 1300 A.D. mentions "Sui Chan" which was "a modern instrument consisting of nine cups, struck with a stick." Japan had the "Hi," a "teacup-shaped porcelain gong." Another record mentions the "Saki Kazat" (earthenware musical bowls with the amount of water in them varied). There are 15th Century Arabic references to musical cups (Kizam) and jars (Khaurabi), and the water content is mentioned. The French *Encyclopedie* credited the Persians with having for "a long time" a method of producing sounds from glasses containing water.

### THE MUSICAL GLASSES IN EUROPE

A *Theory of Music* by Gafori published in Milan in 1492 contains a woodcut showing the playing of glasses in connection with a Pythagorean experiment. In 1596 a Vienna museum catalog describes "ain instrument von glassmerck" that played a range of three-and-a-third octaves, including semitones. A German work printed at Nuremberg in 1677, according to Pohl,[3] gives this recipe for drawing ear-tingling music from wine-glasses:

> To produce a merry wine-music, take eight glasses of equal form; put in the one a spoonful of wine, in the other two, in the third three, and so on; then let eight persons, with fingers dipped in wine, at the same moment pass them over the brim of the glasses, and there will be heard a very merry wine-music, that the very ears will tingle . . .

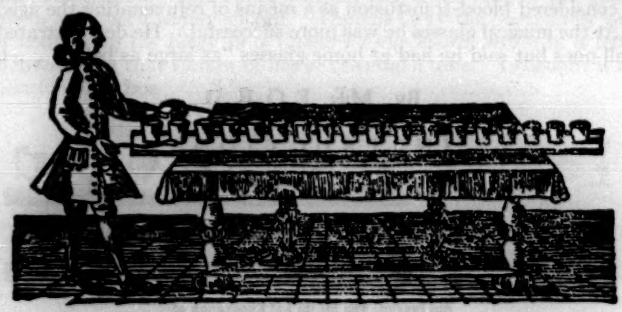
The book also suggests an experiment with four glasses containing brandy, wine, water and salt water (or oil). The different sounds produced by the contrasting content of the glasses were thought to cor-

[2] A. HYATT KING, "The Musical Glasses and Glass Harmonica," *Proc. Royal Musical Assn.*, Session LXXII, April, 1946.

[3] KARL FERDINAND POHL, "Cursory Notices on the Origin and History of the Glass Harmonica," London, Petter & Galpin, 1862. Published for the International Exhibition held in London, 1862.

respond to the emotions aroused by the four "humours," the tradition of body fluids handed down from Hippocrates. King states the diverse sounds were thought "even to have the power of alleviating or curing such disorders as thickness of the blood."

Other references include a musical lexicon published in Germany in 1732, which states that a musician of Silesia used to play concerts on the glasses, accompanied with violins and bass. In France the glasses were known as *Verillons*. The player stood alongside a table with a



The *verillons*, a board with eighteen glasses, from Eisel's *Musicus Autodidactos* (1738).

board that held 18 glasses and, with a slightly curved muffled stick (or two of them), he tapped the glasses on their sides.

### THE MUSICAL GLASSES IN ENGLAND

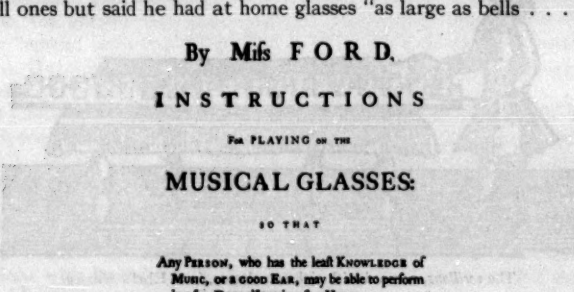
The glasses reached the peak of their fame in the British Isles through the ingenuity and showmanship of Richard Pockrich, an Irishman whom Franklin mentions in his correspondence. Pockrich owned a brewery that failed and, being an excellent amateur musician, followed this bent. He "invented or rather re-invented," says *Grove's*, the Harmonica (Musical Glasses) and gave concerts in Dublin and also toured England during 1743 and 1744. That was thirteen years before Franklin's arrival in England (1757), and several years before Gluck's Haymarket concert.

The harmonica Pockrich "invented or rather re-invented" was, needless to say, not Franklin's instrument. It was a rough predecessor, yet the inspiration of Franklin's invention. Pockrich's was the traditional series of glasses standing upright, with water used to vary the pitch of sound. Some time later Franklin, in his July 13, 1762 letter to John Beccaria of the University of Turin, gives details of Pockrich's arrangement, before going on to describe his own Armonica:

> One Mr. Puckeridge (Pockrich), a gentleman from Ireland, was the first who thought of playing tunes, formed of these tones. He collected a number of glasses of different sizes, fixed them near each other on a table, and tuned them by putting into them water more or less as each note required. The tones were brought out by passing his fingers round their brims . . .

King's study of the glasses brings out interesting facets of Pockrich's life. He calls Pockrich "an adventurer in the best sense of the word" who put his hand to many projects, many of them failing. He planned unsinkable iron ships. He devised wings for human flight. He considered blood-transfusion as a means of rejuvenating the aged.

At the musical glasses he was more successful. He demonstrated on small ones but said he had at home glasses "as large as bells . . . that



By Mifs FORD.

INSTRUCTIONS

For PLAYING on the

MUSICAL GLASSES:

SO THAT

Any PERSON, who has the leaft KNOWLEDGE of MUSIC, or a GOOD EAR, may be able to perform in a few Days, if not in a few Hours.

With CLEAR and PROPER

DIRECTIONS

HOW

To provide a compleat Set of WELL-TUNED GLASSES, at a very moderate Expence.

Miss Ford's *Instructions* for playing the musical glasses.

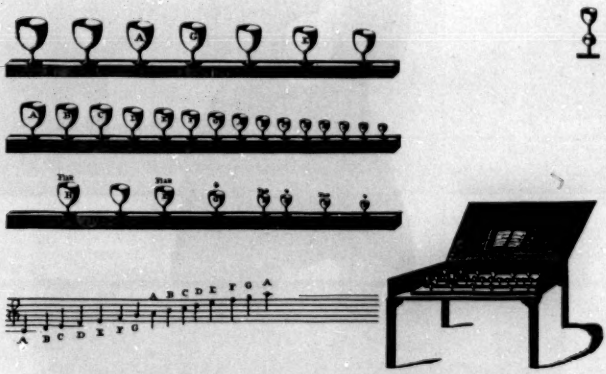
give forth sounds as large as an organ." He called them his "Angelick Organ" and liked to play Handel's "Water Music" on them. For a time he was an attraction at Hamlin's Coffee House near the Royal Exchange in London but in 1759, the second year after Franklin's arrival in England, a fire broke out and Pockrich perished and, with him, his Angelick Organ.

Pockrich's work on the glasses gave them acceptance and status as a fad, and also founded a school of playing. The original Sadler's Wells featured a Miss Wilkinson who performed as a "wire dancer and player of the musical glasses." A Miss Ford, said to be one of Pockrich's pupils, accompanied her own singing on the glasses and wrote a Method. "The Vibration and Tone," she explained to beginners, "is produced by

means of a Certain Degree of Moisture, by wetting the Balls of the Middle Fingers . . . by pursuing the Circular Motion of the Glasses." Her preface states:

> . . . the tones of the Musical Glasses are, from their similitude, more like the human Voice then any musical instrument, that ever was, or, perhaps, ever will be invented . . .

Miss Ford lived to be eighty-six, and survived a stay in Paris during the French Revolution. Her opinion of the rapturous tone of the glasses seems to have been widely shared. Thomas Gray the poet, a few years after composing his *Elegy in a Country Churchyard*, wrote in a letter, "No instrument that I know of has so celestial a tone. I thought it was a cherubim in a box . . ."



Musical glasses according to Miss Ford's *Instructions* of 1761.

**FRANKLIN USED THE HISTORIC PRINCIPLE OF THE MUSICAL GLASSES TO INVENT HIS ARMONICA**

Franklin's coming to England in 1757 introduced him to the glasses when they were at their technically most advanced stage and enjoying their widest vogue. He first saw them at a friend's, a Mr. Delaval, "a most ingenious member of our Royal Society." Delaval had the "completest" set, made "in imitation" of Pockrich's with "a better choice and form of glasses." Franklin's interest in music, his knowledge of science and his instinct for invention worked to suggest developments resulting in the Armonica.

"I wished only to see the glasses disposed in a more convenient form," Franklin afterward wrote, "and brought together in a narrow compass,

so as to admit of a greater number of tones, and all within reach of hand to a person sitting before the instrument . . ."

Franklin's achievement was to unite the glasses in a compact instrument that was convenient to play. His glasses were blown to graded

*Photo by Tana Hoban*

Figurine of Franklin in the costume he wore for his historic appearance before the Lords of the Privy Council, shortly before he returned to America and war broke out between the Colonies and the Mother Country. The costume is in the Massachusetts Historical Society, Boston. The doll, in color, is the work of Helmut Krauhs, of Vienna.

sizes but, instead of standing upright, were turned on their sides and brought together in a concentric arrangement, the rim of one glass overlapping the next. The glasses were mounted on a spindle fixed through their centers. A treadle at the player's feet rotated the spindle "like a spinning-wheel" and the glasses turned. They were wet now and

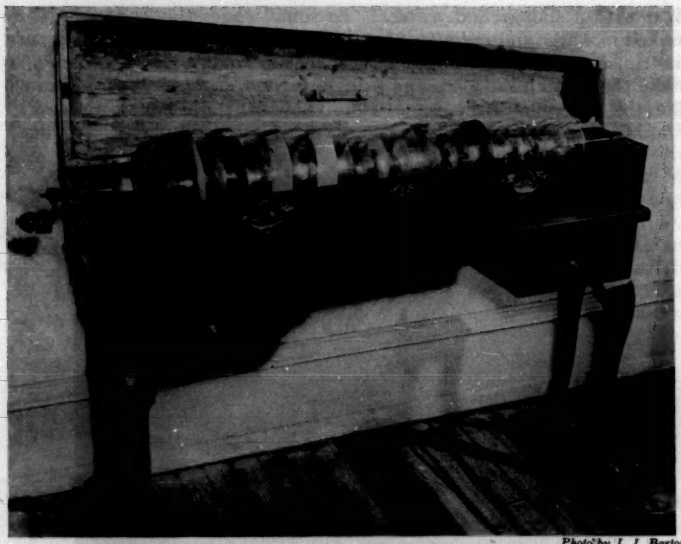
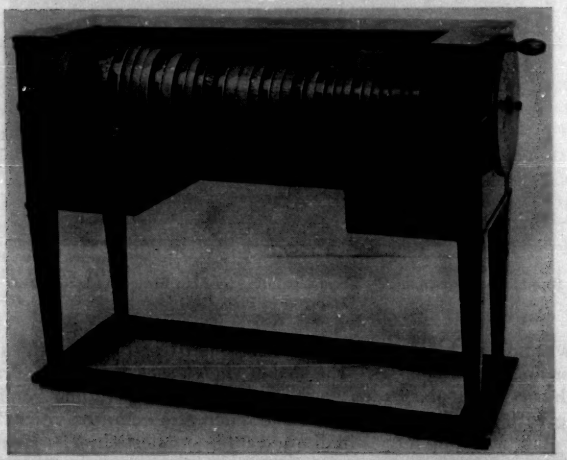


*Photo by J. J. Barlon*

Franklin's armonica, owned by Mr. and Mrs. Wistar MacLaren.



A hand-powered armonica in the Metropolitan Museum of Art in New York. It originally had a treadle which is not attached at present. This model, of European origin, has thirty-five glasses, some having been lost. Franklin specified thirty-seven for his three octaves of sound.

then with a sponge and water.[4] To sound tones the fingers "a little soaked in water and quite free from all greasiness" were pressed against the rims. The fingers might have "a little chalk upon them . . . to make them catch the glass and bring out the tone more readily," much like the action of rosin on a violin bow. "Both hands are used," Franklin wrote, "by which means different parts are played together."

*Courtesy Theodore Presser Co.*

Franklin at his armonica. From a painting by Alan Foster made for *Etude Magazine* in 1926.

It was now no longer necessary to use quantities of water to vary the pitch or note of the glasses—the size of the glasses did that. Franklin used thirty-seven glasses to span three octaves including semitones, starting with the G "a little below the reach of a common voice." His largest glass was nine inches in diameter, his smallest three. He recom-

[4] A later contributor added a trough of water to keep the rims wet.

mended "there should be at least six glasses blown of each size; and out of this number one may probably pick thirty-seven glasses . . . that will be either the note one wants or a little sharper than that note." Thirty-seven different *sizes* did not have to be blown, in other words, because, as Franklin pointed out, "it often happens that two of the same size differ a note or half a note in tune, by reason of a difference in thickness. . . ." The same size might yield several useful tones in the scale.

When testing the glasses for their pitch, the inventor cautioned "to be careful to wipe the glass clean before each trial, because the tone is something flatter when the glass is wet than it will be when dry. . . ." Franklin's glasses were set in a case "about three feet long and eleven inches every way wide within at the biggest end, and five inches at the smallest end; for it tapers all the way, to adapt it better to the conical figure of the set of glasses." The problem of how to distinguish one glass from another when sitting before thirty-seven was settled with Franklin ingenuity. He used color.

> To distinguish the glasses the more readily to the eye, I have painted the apparent parts of the glasses within side, every semitone white, and the other notes of the octave with the seven prismatic colors, viz., C, red; D, orange; E, yellow; F, green; G, blue; A, indigo; B, purple; and C, red again; so that the glasses of the same color . . . are always octave to each other.[5]

On how to perform on the Armonica Franklin said, "This instrument is played upon by sitting before the middle of the set of glasses, as before the keys of a harpsichord. . . ." His own opinion of the tones produced was that they were "incomparably sweet beyond those of any other; that they may be swelled and softened at pleasure by stronger or weaker pressure of the finger, and continued to any length; and that the instrument, being once well tuned, never again wants tuning."

The date of invention has been set at 1762 or 1761, with stronger opinion favoring the latter. Franklin called the instrument the Armonica "in honor of your musical language [Italian]," he wrote his friend-in-science in Turin. Usage—or perhaps more accurately "slippage"—soon made *Harmonica* of Armonica, and Harmonica remained the name. In Germany it was known as the *Glass Harmonica*. In England there were designations like *Glassy-chord* and *Glassichord*.[6]

Franklin's "compleat" instrument which was "never out of tune" got attention rapidly. A London manufacturer made models for 40 guineas. Goldsmith refers to it in *The Vicar of Wakefield* (1766). He remarked the voguish ladies threw the good vicar's daughters "in the shade" because "they could talk of nothing but high life, and high-lived company, with other fashionable topics, such as pictures, taste, Shakespeare, and the musical glasses."

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[5] From Franklin's letter to Beccaria, already cited.

[6] The so-called mouth organ, or harmonica or French harp, is not the Franklin Armonica, but the Aeolina. It was probably invented by Sir Charles Wheatstone (1829), English physicist and pioneer in telegraphy, who is also credited with the Concertina.

The voguish ladies turned out to be Marianne and Cecily Davies, musical artists once reported as relatives of Franklin but, after later study, believed friends only. Marianne had been performing on the musical glasses in public for a decade, and Cecily was a singer. Marianne was given an armonica by its inventor. She became well practiced on the instrument and took it on tour the length and breadth of the land. The *Bristol Journal* of January 12, 1762[7] carried this report:

> The celebrated glassy-chord invented by Mr. B. Franklin of Philadelphia; who has greatly improved the musical glasses, and formed them into a compleat instrument to accompany the voice; capable of a thorough bass, and never out of tune. Miss Davies from London was to perform in the month of January, several favourite airs, English, Scotch and Italian, on the Glassy chord (sic) accompanied occasionally with the voice and German flute.

*Jackson's Oxford Journal* had this a few months later:

> THE ARMONICA. Being the musical glasses without water formed into a compleat instrument capable of a thorough bass and never out of tune, made by Charles James, of Purpool Lane, near Gray's Inn, London. N.B.—The maker is the person who has been employed in the management of the Glass Machines from the beginning, by the ingenious and well-known inventor, which are on the same principles and guided by the same hand as that played on by Miss Davies at Spring Gardens, London, at Bath and Bristol.

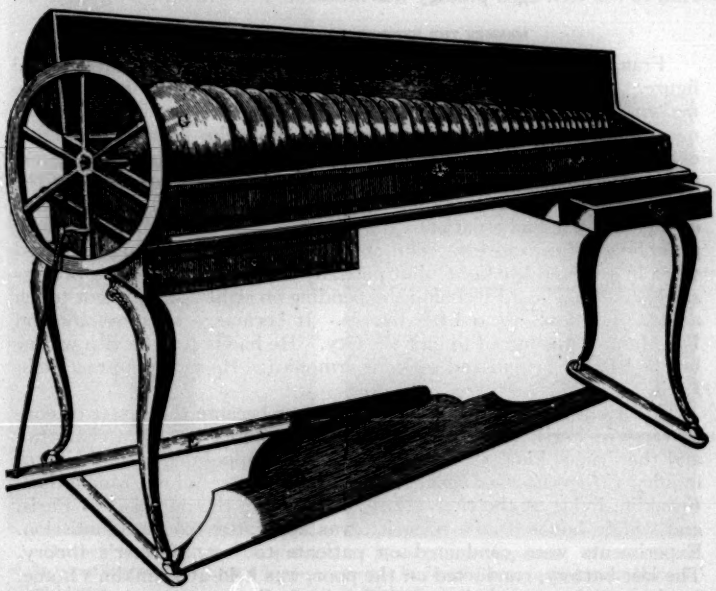
### THE ARMONICA COMES TO AMERICA

Franklin returned to Pennsylvania in 1762. His new instrument crossed the Atlantic, too, and was given a cordial reception in many places. The *Pennsylvania Gazette* of December 27, 1764 carried an advertisement about a "Concert of Music" for "the Benefit of Mr. Forage" consisting of "a variety of the most celebrated Pieces now in Taste, in which also will be introduced the famous Armonica, or Musical Glasses, so much admired for the great Sweetness and Delicacy of its Tone. Tickets at 7s, 6d. each." Washington in 1765 wrote in his diary he had heard the armonica. The *South Carolina Gazette*, in 1774, carried notices of "Signora Castella, who will also perform several airs on the harmonica." A Duetto on the instrument was played in New York in 1793, and the next year John Christopher Moller took part in Philadelphia in the quartet he had written for the harmonica, two tenors and cello. Charleston was the setting in 1795 for "Pot Pourris on the harmonia (sic) by Mrs. Pick." In the same year a "sonata on the Italian Harmonica" was performed in Petersburg, Va. A description of Colonial mansions of the day places an armonica in Nomini Hall, the mansion of Councillor Robert Carter, and says it was designed by "Mr. B. Franklin of Philadelphia, an Armonica, being the musical glasses without water, formed into a complete instrument capable of thorough bass and never out of tune."

[7] The early date in 1762 supports the claim Franklin finished his invention in 1761.

Fithian, a tutor at Nomini Hall, was much taken by the armonica, as shown by his journal entry:[8]

> WEDS. 22 Dec. (1773) . . . Evening. Mr. Carter spent in playing on the Harmonica; It is the first time I have heard the Instrument. The Music is charming! The notes are clear and inexpressibly soft, they swell, and are inexpressibly grand; and either it is because the sounds are new, and therefore pleased me, or it is the most captivating Instrument I have ever head. The sounds very much resemble the human voice, and in my opinion they far exceed the swelling organ.



Before the days of photographs, an Eighteenth Century engraving of Franklin's instrument.

Thomas Jefferson, himself a scientist, musician and inventor, spoke in superlatives on the armonica. He called it "the greatest present which has been made to the musical world this century, not excepting the pianoforte."[9]

### THE ARMONICA ON THE CONTINENT

Marianne Davies, the English performer, played Franklin's instrument in capitals of Europe. Touring with her sister Cecily, she appeared in Paris, Florence, Milan, Turin and brought the armonica to

[8] See O. G. SONNECK, "Suum Cuique (Essays in Music)," New York, Schirmer, 1916, p. 67.
[9] O. G. SONNECK, *op. cit.*, p. 70.

Vienna. In June 1769 she and Cecily performed an "Ode" specially written for the Duke of Parma's wedding, "on the new instrument (l'Armonica) invented by the celebrated Dr. Franklin." In Vienna Marianne taught Marie Antoinette, daughter of the Austrian Empress Maria Theresa, to play. She met Leopold Mozart, farther of the boy wonder, Wolfgang. Papa Mozart undoubtedly played the Davies armonica and wrote in a letter "Wolfgang too had played upon it." And so the teen-aged prodigy was introduced to Franklin's instrument.

### MESMER THE HYPNOTIST USED AN ARMONICA

Franz Anton Mesmer was a doctor of Vienna, and a controversial figure. His name is associated with the mysteries of Animal Magnetism and hypnotism. He played Marianne's armonica, then had one made for himself. It was said he used the armonica's highly emotional tones to bring on a receptive state in his patients. His belief that a life force was transmitted from the planets through him in the form of Animal Magnetism had already roused the medical profession of Vienna. He had a patient called Marie Paradis, a blind pianist and godchild of the Empress. The antagonism toward Mesmer mounted when he restored her sight—but partially only. In the healing process she lost her keyboard technique, depending on sight now, and not touch alone. The loss affected her nerves. It became a *cause célèbre* and Dr. Mesmer was forced to quit the city. He had married a rich widow but he left her behind and took his armonica. He set up a practice in Paris.

A curious aftermath is that Dr. Mesmer became the center of controversy in Paris, too. The profession denounced him as a charlatan and the French king, the hapless Louis XVI, appointed a comission of inquiry (*Magnétiseme Commission*) to investigate Animal Magnetism. Franklin, living at the time (1784) in Passy on the outskirts of Paris, and widely honored as a scientist, was appointed to the commission. Experiments were conducted on patients to test Mesmer's theory. The last battery, conducted on the poor, was held at Franklin's home. In the end the commission, reporting its findings, admitted that cures had been made but "has unanimously come to the conclusion that there is nothing to show that the fluid of Animal Magnetism exists . . ." Mesmer, a player of the armonica, was discredited by its inventor. He left Paris and after he retired continued to find pleasure and consolation playing his armonica.

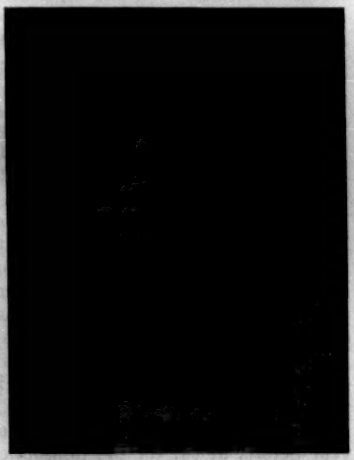
### MOZART'S MUSIC FOR THE ARMONICA

At about the time Marianne Davies introduced the armonica to the boy Mozart, a girl named Marianna Kirchgessner was born in Baden, Germany (1770). Tragically, when four, she lost her eyesight and, compensating, she exploited a natural aptitude for music and became

expert as a player of the *glass harmonica*. The composer Naumann said she was "unrivaled." She appeared in important musical centers of Europe and in London.

Wolfgang, as a boy wonder, had made the name Mozart ring through Europe. When he was only six, his father had started him with his elder sister, a musician too, on the concert circuit.[10] He had made triumphal tours of Austria, Hungary and Germany and had visited

The boy Mozart. A bronze statue of the versatile prodigy by Ernest Barrias, in the Louvre.



Mozart at twenty-seven. An unfinished portrait by Joseph Lange, in the Mozart Museum in Salzburg.

Brussels, Paris and London with acclaim. In Italy he developed a strong love of opera. The great composer "Papa" Haydn told Papa Mozart he considered "your son the greatest composer I have ever heard."

Mozart, now in his mid-thirties, had produced volumes of music with many immortal masterpieces among them. His compositions appeared in various forms, depending on how they were to be used or where played. For *Voice* there were songs and arias; for *Instruments*, concertos and sonatas for piano, violin, flute, etc.; for *Small Concert Groups*,

[10] W. J. TURNER, "Mozart: The Man and His Works," Garden City, Doubleday, Anchor Edition, 1954.

many combinations of quartet, quintet, etc.; for *Orchestra*, symphonies, divertimentos, serenades; for *Church*, cantatas, canons, masses; for the *Opera*, lasting favorites like *Marriage of Figaro* and *Don Giovanni* with *The Magic Flute* soon to be written. Mozart was brought up a Roman Catholic and made a Chevalier by the pope when fourteen but, like Franklin, had become a Freemason and in fact wrote special Masonic music.[11]

About twenty years had passed since Mozart first heard Franklin's armonica in musical Vienna and now (1791) he witnessed the spectacle of the blind Marianna Kirchgessner playing the instrument at a concert. He was enthralled with her performance and composed, for her, an *Adagio and Rondo*. It was a quintet to be played by Glass Harmonica, Flute, Viola, Oboe and Cello. It bears the catalog number 617-A.[12] Alfred Einstein, the eminent musicologist, made this comment about the music[13]:

> The instrument played by the blind virtuoso was not quite the same as the somewhat primitive one of Franklin; it must have had a keyboard and there must have been added to it later, as Gerber reports in his *Neves Lexicon*, an elastic sounding board. Again, Mozart was very limited in his use of the lower portion of the gamut, for the instrument did not go below G in the alto register. Nevertheless, this is one of the "heavenly" works . . .

Mozart also wrote an *Adagio for Harmonica* Solo (K-356, later K-617a). Einstein says Marianna took both these "pieces all over Europe and apparently had particular success with them in London."[14] When giving the second piece its premiere performance, in Vienna, she announced in the *Weiner Zeitung* that Mozart would accompany her, supposedly on the viola. If this actually happened, "it must have been," King comments, "one of the very last occasions on which he played in public." Mozart, ailing, overworked and despondent, died that year, when only 35.

Franklin never heard the music Mozart wrote for his armonica. The year before, at 84, the old philsopher and "sage that two worlds acclaim," the dean of achievement, the musician and inventor had shown the younger man how to put on the mangle of immortality.

### A NOTE ON THE HARMFUL EFFECTS OF PLAYING THE ARMONICA

We noted earlier that in the 17th Century the musical glasses were considered to have an effect in therapy. Some even thought it would

[11] *The Magic Flute* was "a symbolic glorification of Freemasonry," says the Columbia Encyclopedia.

[12] Kochel arranged Mozart's complete works into 626 items or pieces. Thus each number has a "K" designation following it.

[13] ALFRED EINSTEIN, "Mozart: His Character and His Work," Oxford, Oxford University Press, 1945.

[14] Mozart began a second Quintet for Harmonica with the same accompanying instruments. The fragments (Pohl wrote in 1862) are "to be seen among the sketches of Mozart in the Mozarteum at Salzburg."

cure "thickness of the blood." King, in mentioning the "astonishing effect" the armonica had on German life, has this to say about its potency: "Its tones could reconcile quarrelling friends; restore fainting men to consciousness; make women faint; send a dog into convulsions; make a sleeping girl awake screaming . . . and even cause the death of one very young."

Mesmer certainly used the instrument's delicate strains in treating patients. Among players, the vibrations of sounds reacting on the fingertips were said to bring on a nervous state. Naumann the composer and Marianna Kirchgessner (for whom Mozart wrote his pieces) both had to give up playing. O. G. Sonneck, in his musical memoirs, mentions this in connection with Marianne Davies, Franklin's English friend who brought the armonica to Vienna. "Gradually Marianne's nerves became so seriously affected by performance on the armonica (so frequently as result of continued performance on the instrument as to have occasioned official prohibition of its use in many Continental towns), that she was compelled to retire from her profession. She died in 1792" (the year after Mozart).

The subject is also dealt with by Charles Ferdinand Pohl, who wrote many years before Sonneck. Pohl was a musician of Vienna, a player of the armonica who came to London for the International Exhibition in 1862. For the Exhibition he wrote a pamphlet for visitors who "will for the first time have an opportunity to see an almost forgotten instrument—the Glass Harmonica." Pohl was the grandson of Ferdinand Pohl, a "simple joiner" of Bohemia who made glass harmonicas. The grandson says this in his Exhibition pamphlet:

> Notwithstanding the objections made against the instrument, as having a tendency to affect the nerves—indeed, so much so as to cause it to be forbidden in several countries by the police (in the museum at Salzburg it is still [1862] shown to the visitors as such) —its introduction to the public, and its consequent favor could not be prevented; and even the poets of that period sang its praises for its sweet and melting tone, and this tended to render it popular.

"Attempts have been made to improve or modify the harmonica," says *Grove's Dictionary of Music and Musicians*, "by substituting a violin bow for the hand, or by reducing the peculiarly penetrating and exciting tone which is said to be prejudicial to the nerves of the players—but without success." Pohl explains this lack of success by remarking that the device "which should substitute the application of the hand was too soon worn out, and, consequently, the peculiar and ethereal tone, which alone could be produced by the application of the finger, was lost." The same chronicler quotes Fr. Rochlitz, once well known in musical circles, who (1798) wrote in defense of the armonica:

> I maintain, from experience, that the effect of its play is as little injurious as that of any other expressive instrument. In the choice of compositions best adapted for it, one can easily select such pieces as touch the heart and softly animate its feelings.

> But, be it observed, the Harmonica, like all soft and delicate instruments, should not be too continuously played; like every pleasure of a refined nature that leads the mind to contemplation, if too much enjoyed, it abstracts us from the ordinary enjoyments of social life, and thus may become an evil. Also, we should not indulge in its soft and pleasing tones too much at night. These are the only precautions I deem it necessary to recommend in playing the Harmonica—a precaution which we must exercise in all that excites the feelings in a higher degree, however it may raise, elevate, and refine the taste and sentiment.

This advice may have been heeded by armonica players in general, but Dr. Mesmer was an exception. He liked to play the instrument, it is reported, "at twilight, after dinner."

### THE ARMONICA'S VOGUE GRADUALLY FADED

It was played in England and America, as we have seen, and in Germany it came along on the high tide of romanticism, which gave it free expression. "Of all musical inventions," said a German newspaper, "the one of Mr. Franklin has created perhaps the greatest excitement. Concerning the way of producing tones, it is an entirely new kind of instrument." But apart from Marianne Davies' performances in Austria and Italy there is not much evidence, King says, that "it or the glasses (as distinct from the harmonica) were known in Europe generally." A model of the armonica built in Bohemia and shown at the London Exhibition of 1862[15] was described as "an almost forgotten instrument." (In fact, half a century earlier (1795), and only five years after Franklin's death, John Christopher Moller in America had played the armonica at a concert to "introduce that instrument . . . of which the late Dr. Franklin was the inventor.") Its vogue had passed because the instrument had to be made to order by hand and wasn't cheap,[16] because it needed practicing to play satisfactorily and because its possible injurious effects made people cautious.

Other music, besides Mozart's, was written for the armonica. Beethoven wrote a few bars for it for his opera *Leonora Prohaska* (1814). Naumann, a good hand at playing the armonica, wrote six sonatas for it. Moller composed his quartetto. When Mozart's blind player died, Tomaschek wrote a *Fantasie on the Grave of Miss M. Kirchgessner*.

Improvements were tried over Franklin's original, generally in the interest of bettering its playability. Mazzuchi had one played with a violin bow, its tones being "less exciting" to persons of weak nerves. Hesel in 1785 had a key harmonica called the *clavier-harmonica*. Rollig in 1786 also had a key instrument and visited so many glass houses in

[15] The armonica displayed at the London Exhibition of 1862 was made in Bohemia by Emmanuel Pohl, "a manufacturer of this instrument." It is believed to be the "Glass Harmonica, Modern" that was later purchased by the South Kensington Museum for fifteen guineas, now in the Science Museum, London (in its store at Sydenham). The supposition is corroborated by the 10th Edition of the *Brittanica* (1910) which stated, "One specimen by Emmanuel Pohl, a Bohemian maker, is preserved in the Victoria and Albert Museum, London" (sister museum of the Science Museum).

[16] Papa Mozart said Dr. Mesmer's "beautifully made" harmonica cost fifty ducats.

Hungary, Bohemia and Germany that people thought he was the original inventor. Francis Hopkinson, the American composer, wrote Thomas Jefferson that he had "applied keys to the Glasses, furnished with artificial Fingers." He admitted to partial failure because "it required too much Address in the manner of wetting the Cushions for Common Use." Einsten said, as previously noted, Mozart's music must have been written for an instrument with a keyboard. Boyer, in Paris, invented something Franklin named the *Glaschord*. Kaufman in Dresden had a *Harmonicon*, Chladny a *Euphon* and a *Clavicylinder*. The Britannica cites a *Melodicon* in 1800 and a *Panmelodicon* in 1810.

A modern version of Franklin's armonica reproduced by the American Academy of Arts and Sciences and the Franklin Savings Bank, of Boston, and built by Herman Schlicker of Buffalo, with the glasses blown by Corning. Seated are (left to right); Maynard L. Harris, President of Franklin Savings; John E. Burchard, President of the Academy; and E. Power Biggs, organist. Mr. Biggs played both the musical glasses (not shown) and the power-driven armonica at a concert April 11, 1956, in the Kresge Auditorium of the Massachusetts Institute of Technology in Cambridge.

The most recent attempt at building a Franklin armonica, and doubtless the only one in the Twentieth Century, was undertaken recently by the American Academy of Arts and Sciences with the Franklin Savings Bank of Boston. A replica of Franklin's armonica, driven by motor, was built with the cabinet and keyboard mechanism the work of Herman Schlicker, Buffalo organ builder. The glasses were blown by Corning Glass Works. The armonica was presented to the public as "an instrument in transition" on April 11, 1956 in the Kresge Auditoritum at Massachusetts Institute of Technology. E. Power Biggs, the eminent organist, performed on the armonica and also played the muscial glasses.

In Franklin's time there were many players of the armonica, but none has come down with the authority of the two Mariannas who linked the names of the two immortals—Marianne Davies who introduced the armonica to Mozart and Marianna Kirchgessner, the blind player who inspired the master to compose for the instrument.

### FRANKLIN AND MUSIC

The simple songs and "psalm tunes" Ben grew up on gave him early kinship with music. His father, a candle maker, was "skilled a little in music," Ben wrote in his *Autobiography*. The boy wrote ballads in the Grub Street Style (*Lighthouse Tragedy*). Other songs with Franklin words are *Sailor Song*, *My Plain Country Joan* (praising his wife), *Fair Venus Calls* and *The Mother Country* (on his second visit to England). Grounding in music prompted him to grasp the potential of the musical glasses. Experience playing the violin, the guitar and the harp gave him fundamentals and fingering to play his own new instrument.

Music was a lifetime hobby that commanded attention in his crowded panorama of interests. "Nothing he wrote between 1757 and 1762, no experiment he undertook and carried out absorbed him more happily," writes Carl Van Doren, "than his musical invention." After he gave a performance on the armonica in America, a rapt listener[17] sent him this fond effusion:

> Hark! the soft warblings, sounding smooth and clear,
> Strike with celestial ravishment the ear,
> Conveying inward, as they sweetly roll,
> A tide of melting music to the soul;
> And sure if aught of mortal moving strain,
> Can touch with joy the high angelic train,
> 'Tis this enchanting instrument of thine,
> Which speaks in accents more than half divine.[18]

[17] Nathaniel Evans, early American poet. The above is the last stanza of his poem "To Benjamin Franklin, Esq., LL.D., Occasioned by Hearing Him Play on the Harmonica."

[18] In the chorus of praise for the armonica's sweet and beautiful tones there seems to be only one discordant note. "Physicians speak also of a 'Burning Harmonica'," Pohl wrote. But this was not inappropriate, he explained, because "individuals who have been present at the burning of the gas of hydrogen are well acquainted with the singular tone produced by this operation." Doubtless it was music to scientific ears!

Franklin knew enough about music to write a "Criticism of Musick." He didn't approve of what he called the "modern music" of his time. He considered melody a "succession of sounds" and harmony "only the coexistence of agreeable sounds." Scotch songs found favor because they properly united melody and harmony. The impact music made is shown in his listing of the emotions music can express, such as, Desire, Joy, Grief, Complaint, Expostulation, Resignation, Courage, Contempt, Tranquillity, Reverence, Resentment, Caution, etc.

There is little evidence that Franklin was a serious composer. In 1941 a string quartet bearing his name ("Benj. Francklin," the copyist wrote it) turned up in the library of the Paris Conservatory, but this seems to have been written in the spirit of fun. It was scored for three violins and cello, and written so the bows played "open" strings only, with no action demanded of the playing fingers.[19] It didn't take skilled players to toss it off.

When, as Minister, Franklin lived in Passy, then a village outside Paris, he took part in musical soirees in the homes of fashionable neighbors. The captivating Mme. Brillon de Juoy who called him "mon cher papa" was a favorite and an accomplished musician and composer. Boccherini dedicated six sonatas to her. Among her own compositions was the *March of the Insurgents*, a tribute she wrote at the joyful news the Americans had defeated Burgoyne at Saratoga. This confiding lady was Franklin's junior by decades. The flirtatious admiration that charged the atmosphere between them sparked many a pleasantry, spoken or written. Franklin, in a flattering promise to wait forty years for her in Paradise, gallantly wrote, "I shall have plenty of time . . . to practice the harmonica, and maybe I shall then be worthy of accompanying your pianoforte. Now and then we shall give little concerts . . ."

Franklin said in a letter that "Mr. Pagin . . . is assuredly one of the best men possible, for he had the patience to listen to me playing an air on the armonica, and to hear it to the end." He was referring to "Old Pagin," one of the Passy musical group who, Madame promised in a letter to *mon cher papa*, "would play the *God of Love* on the violin, I the march on the piano and you [Franklin] *Little Birds* on the harmonica." She also mentioned Dr. Mesmer who "will be contented with playing on the harmonica without boring us with electric fluid."

Thus the man of many interests drew inspiration from music, and used music to bring inspiration to others—a lifelong ambition. When Franklin died he bequeathed his instruments to his daughter Sarah's husband, Richard Bache. The inventory listed them as a harmonica, a spinet, harpsichord, glassichord and a "viol de gambo & bells."

[19] The music is marked *Scordatura* ("mis-tuned"), a method of tuning stringed instruments so that many harmonies might be played easily just by drawing bows across the open strings. The scordatura was not unusual among compositions for violin, and was frequent in Scottish reels.

In 1926, when the 150th anniversary of American independence was being celebrated, the music trades of the United States remembered what most Americans may have forgotten or never known—that the "Patriot" of many accomplishments was also accomplished in music. At their annual convention, they unanimously proclaimed the inventor of the armonica "Patron Saint of the Music Industries."

## APPENDIX

*Recent Performances, Music and Records*

*Performances*

For years the Armonica has lain, unplayed and without players, in homes and museums—"an almost forgotten instrument," as Pohl described it in 1862. Mozart's Quintet was performed at the London Museum in 1938, with the Armonica part played by Bruno Hoffmann of Stuttgart on the Glass Harp, an instrument "of his own construction, independent of Franklin." It was also played in 1951 at a Children's Concert of the Philadelphia Orchestra, the Celesta taking the Armonica part. E. Power Biggs, organist, played the Adagio for Harmonica (solo) at the Cambridge Concert of the American Academy of Arts and Sciences in April, 1956.

Two movements of Benjamin Franklin's String Quartet were performed in February, 1955 at another Children's Concert of the Philadelphia Orchestra, by students of the Settlement Music School.

*Music*

Adagio and Rondo, K-617 (Quintet)—Mozart

An edition with parts for all instruments is published by International Music Company, New York. The Armonica part is scored for the piano.

Mercury Music Company, New York, also has an edition.

*Records*

Adagio and Rondo, K-617 (Quintet)—Mozart

Vox has recorded this chamber work, with the Pro Musica Chamber Orchestra of Vienna as artists. The Armonica part is played on the Celesta. VOX PL 8550.

A recording has also been made by Audiophile Records (Saukville, Wisconsin), a brand of limited distribution among "hi-fi" enthusiasts. The records are considered by some to be the most technically perfect made. The Los Angeles String Quartet, with a Harpsichord taking the Armonica part, plays the Quintet. Audiophile AP-22.

Adagio for Glass Harmonica (solo), K-617a.—Mozart

Played by E. Power Biggs, organist, as one of various selections on the record. The soft flute stops of the organ are used to simulate the ethereal tones of the Armonica. Columbia ML 4331.

# THE APPLICATION OF MATRIX METHODS TO MULTI-VARIABLE CONTROL SYSTEMS

BY

R. J. KAVANAGH [1]

ABSTRACT

The methods of matrix algebra are used for the description, analysis and synthesis of linear multi-variable control systems. Practical examples of the analysis and synthesis of systems are given and criteria for the realizability of multi-variable control systems with desired characteristics are derived.

## 1. INTRODUCTION

This paper presents some basic mathematical ideas which may be used to analyze and synthesize multi-variable control systems. Such systems are those in which one or more output variables are controlled by one or more input variables. This class of systems includes as a special case the common single variable control system.

Because of the greater number of degrees of freedom implicit in a system involving the control of several variables, the analysis or synthesis of such systems becomes considerably more complicated than is the case with single variable systems. Since the mathematical description of multi-variable systems, in the linear case, involves a set of simultaneous equations in the system variables, the possibility of using matric methods to solve analysis and synthesis problems becomes apparent. This, in fact, is the approach that has been used in this paper. The advantages of the matric approach have been of two kinds. In the first place the algebra involved in the solution of problems is considerably condensed and in the second place, it has been possible to draw on some ideas of matrix algebra to evolve analysis and synthesis techniques for multi-variable systems.

It is appropriate at this point to mention a few examples of multi-variable control systems which are present in the engineering field. As an example of a system with many variables, the steel rolling mill may be cited. Here input variables are quantities such as steel input gauge, rolling speed, distance between rollers and roller torque. Output quantities include the steel output gauge and the steel tensions between the stands. A less complicated example is furnished by the turbo-jet engine. In this case the inputs are rate of fuel flow and exhaust nozzle area. The output variables are the engine speed and temperature. Further examples are given by various chemical processes where the quality of the output commodity depends on the accurate control of

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several input variables. In all multi-variable systems such as those given above, the problem of control is that of regulating the input quantities in such a way that the output variables will behave in the desired manner. The technique of feedback may or may not be used for this purpose.

Previous studies made in the field of multi-variable control systems have been restricted to particular cases of multi-variable systems and no general theory has yet appeared. It is hoped that the work described in this paper will partially remedy this lack. Matrix theory was first applied to multi-variable systems although only in a limited sense to a particular problem, by Boksenbom and Hood (1).[2] They dealt with the problem of controlling a turbo-jet engine. A more recent paper by Cruickshank (2) gives a basis to the matrix analysis of multi-variable control systems but still without complete generality. Other papers which have been concerned with the properties of multi-variable systems have been written by Golomb and Usdin (3), Povejsil and Fuchs (4), Pack and Phillips (5), and Raymond (6).

In this paper, the basic matric equations for the general linear multi-variable control system will first of all be derived and stated in the forms useful for analysis and synthesis. Several examples of design and analysis of control systems will then be given. Finally the conditions for realizability of desired systems will be derived.

## 2. BASIC EQUATIONS

A block diagram of a multi-variable system is shown in Fig. 1. The system $H$ has $m$ output variables, $y_1, y_2, \ldots y_m$ which are each dependent upon the $n$ input variables $z_1, z_2, \ldots z_n$. The manner in which the $y$'s are related to the $z$'s determines whether the system is linear or non-linear. In the linear case, the equations describing the system in the steady state will be of the following form:

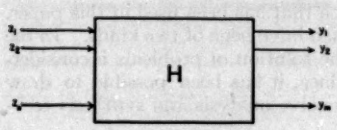


FIG. 1. General multi-variable system.

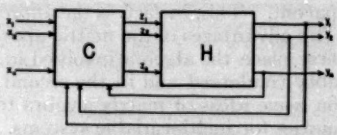


FIG. 2. General linear multi-variable control system.

$$
\begin{aligned}
y_1 &= h_{11} z_1 + h_{12} z_2 + \ldots + h_{1n} z_n \\
y_2 &= h_{21} z_1 + h_{22} z_2 + \ldots + h_{2n} z_n \qquad (1) \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
y_m &= h_{m1} z_1 + h_{m2} z_2 + \ldots + h_{mn} z_n
\end{aligned}
$$

[2] The boldface numbers in parentheses refer to the references appended to this paper.

where the $y$'s, $z$'s and $h$'s are functions of the complex frequency variable.

In the non-linear case it is not possible to write such a set of linear equations. However, by considering incremental variations in the $y$'s and $z$'s about a fixed operating point, a corresponding set of equations may be written. Hence, in this paper, the properties of linear systems, or non-linear systems operating in the region of a given operating point, will be described. No attempt will be made to deal with general non-linear operation.

The set of equations (1) completely describes the block $H$ in a way that is analogous to the usual transfer function description of a single input-single output system. Using matrix notation, these equations may be written in condensed form:

$$Y = HZ \tag{2}$$

where $Y$ is an $m \times 1$ column matrix, $H$ is an $m \times n$ matrix and $Z$ is an $n \times 1$ column matrix. Thus $H$ is given by:

$$H = \begin{vmatrix} h_{11}\, h_{12} \ldots h_{1n} \\ h_{21}\, h_{22} \ldots h_{2n} \\ h_{m1}\, h_{m2} \ldots h_{mn} \end{vmatrix}. \tag{3}$$

This matrix, whose elements are functions of the complex frequency variable, completely describes the multi-variable system.

The control of a multi-variable system is effected in the general linear case by making the $z$'s linear functions of the over-all system inputs and outputs. Figure 2 shows a diagram of the most general linear method of controlling a multi-variable system.

Block $C$ is a computer such that each variable $z$ is a linear combination of all the $x$'s and all the $y$'s. The $x$'s are the over-all system inputs and by means of $C$ it is desired to obtain a suitable relationship between the $y$'s and the $x$'s.

The computer $C$ may be described by the following set of linear equations:

$$\begin{aligned} z_1 &= c_{11} x_1 + c_{12} x_2 + \ldots + c_{1n} x_n + k_{11} y_1 + k_{12} y_2 + \ldots + k_{1m} y_m \\ z_2 &= c_{21} x_1 + c_{22} x_2 + \ldots + c_{2n} x_n + k_{21} y_1 + k_{22} y_2 + \ldots + k_{2m} y_m \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ z_n &= c_{n1} x_1 + c_{n2} x_2 + \ldots + c_{nn} x_n + k_{n1} y_1 + k_{n2} y_2 + \ldots + k_{nm} y_m \end{aligned} \tag{4}$$

where the $x$'s, $z$'s, $y$'s, $c$'s and $k$'s are functions of the complex frequency variable. These equations may also be written in matrix form:

$$Z = C X + K Y, \tag{5}$$

where $C$ is an $n \times n$ matrix, $K$ is an $n \times m$ matrix, and $X$ is an $n \times 1$ column matrix. Thus $C$ and $K$ are given by:

$$C = \begin{vmatrix} c_{11}\, c_{12} \ldots c_{1n} \\ c_{21}\, c_{22} \ldots c_{2n} \\ \ldots\ldots\ldots \\ c_{n1}\, c_{n2} \ldots c_{nn} \end{vmatrix} \tag{6}$$

$$K = \begin{vmatrix} k_{11}\, k_{12} \ldots k_{1m} \\ k_{21}\, k_{22} \ldots k_{2m} \\ \ldots\ldots\ldots \\ k_{n1}\, k_{n2} \ldots k_{nm} \end{vmatrix}. \tag{7}$$

These two matrices, whose elements are functions of the complex frequency variable, completely describe the computer.

It is of great interest to determine the relationship between the over-all system input and output variables, that is, between the $x$'s and the $y$'s. This may be done by combining Eqs. 2 and 5.

$$Y = H\,C\,X + H\,K\,Y. \tag{8}$$

It follows that

$$Y = (I - H\,K)^{-1}\,H\,C\,X, \tag{9}$$

where $I$ is the $n \times n$ identity matrix. This equation gives, in matrix form, the relationship between the input and output variables for the over-all system. Equation 9 may be written in shorter form

$$Y = H'\,X, \tag{10}$$

where $H'$ is a matrix describing the over-all system. Hence there results the relationship

$$H' = (I - H\,K)^{-1}\,H\,C \tag{11}$$

from which it follows that

$$H' = H\,C + H\,K\,H'. \tag{12}$$

Equations 11 and 12 are important results. They are two forms of the same equation which relates the transfer matrices of the multi-variable system with and without the controlling computer. Equation 11 is in fact the more general case of the relationship between the open and closed loop transfer functions of a simple servomechanism.

Each term of Eq. 12 is an $m \times n$ matrix. Thus this equation actually represents $mn$ simultaneous equations. For future purposes it will

be useful to be able to write out these $mn$ equations explicitly. This may be accomplished by noting that the $ij^{th}$ element of the matrix on the left of Eq. 12 is equal to the sum of the $ij^{th}$ elements of the two terms on the right hand side. The $ij^{th}$ element on the left hand side is $h'_{ij}$. For the first term on the right, the $ij^{th}$ element is

$$\sum_{g=1}^{n} h_{ig} c_{gj}.$$

The $ij^{th}$ element of the second term on the right hand side is

$$\sum_{l=1}^{m} \sum_{g=1}^{n} h_{ig} k_{gl} h'_{lj}.$$

Thus the $ij^{th}$ equation is

$$h'_{ij} = \sum_{g=1}^{n} h_{ig} c_{gj} + \sum_{l=1}^{m} \sum_{g=1}^{n} h_{ig} k_{gl} h'_{lj}. \tag{13}$$

The necessary equations describing a multi-variable control system have now been derived. The method of using these equations for the synthesis and analysis of systems will now be given.

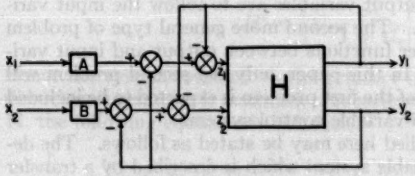


FIG. 3. A two input–two output control system.

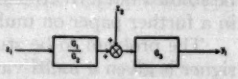

FIG. 4. The basic two input–one output system.

### 3. APPLICATION OF BASIC EQUATIONS

The analysis of multi-variable systems may best be carried out by the use of Eq. 11,

$$H' = (I - HK)^{-1} HC. \tag{11}$$

The problem of analysis consists of determining $H'$ when $H$, $C$ and $K$ are given. An example will make this clear.

Consider the system shown in Fig. 3. This system is a hypothetical two input–two output control system where the main transducer $H$ is described by

$$H = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix}.$$

By inspection of the system configuration it may be seen that the computer is described by

$$C = \begin{vmatrix} A & 0 \\ 0 & B \end{vmatrix}$$

$$K = \begin{vmatrix} -1 & -1 \\ -1 & -1 \end{vmatrix}.$$

Substitution of the $C$, $K$ and $H$ matrices into Eq. 11 yields (after simplification):

$$H' = \begin{vmatrix} \dfrac{A(h_{11} + h_{11}h_{22} - h_{21}h_{12})}{D} & \dfrac{B(h_{12} + h_{12}h_{21} - h_{11}h_{22})}{D} \\ \dfrac{A(h_{21} + h_{21}h_{12} - h_{11}h_{22})}{D} & \dfrac{B(h_{22} + h_{22}h_{11} - h_{12}h_{21})}{D} \end{vmatrix},$$

where $D = (1 + h_{11} + h_{12})(1 + h_{21} + h_{22}) - (h_{11} + h_{12})(h_{21} + h_{22})$. Hence the response of $y_1$ and $y_2$ to inputs $x_1$ and $x_2$ may be determined.

Compared with the problem of analysis, the synthesis of control systems is considerably more complicated. Basically there are two types of synthesis problems. The first is perhaps the true control problem in that it is concerned with the design of a duplicating system, that is, one in which the output variables are to follow the input variables as closely as possible. The second more general type of problem is that in which the transfer functions between output and input variables are to be designed. In this paper, only the second problem will be studied but a treatment of the first problem is expected to be included in a further paper on multi-variable control systems.

The problem to be studied here may be stated as follows. The designer is given a multi-variable system which is described by a transfer matrix $H$. The desired system is required to have a transfer matrix $H'$. The design problem is therefore to derive by means of Eq. 12 a computer which will provide the desired system. The computer is specified by the two unknown matrices, $C$ and $K$ in Eq. 12. As mentioned in the next section, synthesis of a desired system may require a computer which is not physically realizable although it may be mathematically realizable. In such cases the designer must modify the desired system so that a practical computer is possible. Thus in many design problems, the desired system may not be explicitly given and it is necessary to synthesize a system which will not only suit the requirements of the situation but which is also physically realizable.

The determination of the elements of $C$ and $K$ must be made by solving the $mn$ simultaneous equations represented by Eq. 12. The numbers of elements in $C$ and $K$ are $n^2$ and $mn$, respectively. Hence Eq. 12 represents a set of $mn$ equations in $n^2 + mn$ unknowns. In

order that a solution of these equations may be obtained, $n^2$ constraints on the system must be added. These may be already implied in the desired system or they may be arbitrarily prescribed. Once these constraints have been specified, the $mn$ equations may be solved for the remaining $mn$ unknown quantities.

Two examples of the design of multi-variable control systems will now be given. Equation 13 will be used to obtain the simultaneous equations represented by Eq. 12.

*Single Variable System with Disturbance*

A system known as the conditional feedback system has been evolved by Lang and Ham (7) which enables feedback to be used around a given system in order to change the system disturbance response while leaving the input-output response unchanged. This system may be logically designed as follows using the principles given above.

The given system is shown in Fig. 4 and its transfer matrix is

$$H = |G_1 \; G_2|. \tag{14}$$

In Fig. 4, $z_1$ is the primary input and $z_2$ is a disturbance input. The desired system is given by Eq. 15.

$$H' = \left| G_1 \frac{G_2}{1 + G_1} \right|. \tag{15}$$

Hence the system response to the primary input is unchanged but the disturbance response is modified. Substituting Eqs. 14 and 15 into Eq. 12, the following equation is obtained:

$$\left| G_1 \frac{G_2}{1 + G_1} \right| = |G_1 \; G_2| \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} + |G_1 \; G_2| \begin{vmatrix} k_{11} \\ k_{21} \end{vmatrix} \left| G_1 \frac{G_2}{1 + G_1} \right|.$$

This equation represents two simultaneous equations in six unknowns. Four constraints arise from the fact that $z_2$ is a disturbance quantity and so cannot be operated on in any way. Reference to Eq. 4 shows that these constraints are:

$$c_{12} = 0 \qquad c_{22} = 1$$
$$c_{21} = 0 \qquad k_{21} = 0.$$

The simultaneous equations for the system now become

$$G_1 = G_1^2 \, k_{11} + G_1 \, c_{11}$$

$$\frac{G_2}{1 + G_1} = G_2 + \frac{G_1 \, G_2}{1 + G_1} \, k_{11}.$$

The unique solution of these equations is

$$c_{11} = 1 + G_1$$
$$k_{11} = -1.$$

Hence, substituting in Eq. 4,

$$z_1 = x_1 (1 + G_1) - y_1$$
$$z_2 = x_2.$$

Two possible configurations which satisfy these equations are given in Fig. 5. These two systems both have the transfer matrix given in Eq. 15.

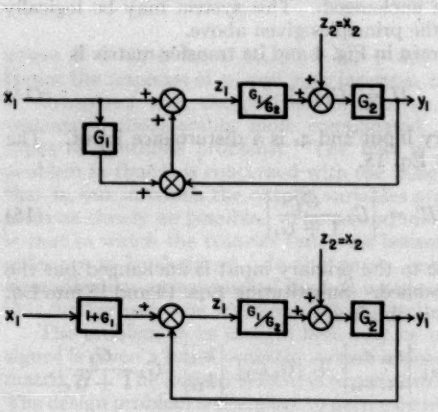


FIG. 5. Two possible forms of the final two input–one output control system

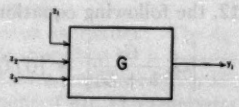


FIG. 6. The basic three input–one output system

This design problem is notable for the fact that the solution of the equations for the computer elements is unique. An example in which this solution is not unique will now be given.

*Two Input–One Output System with Disturbance*

The initial system is shown in Fig. 6. In this diagram $z_1$ is a disturbance input while $z_2$ and $z_3$ are the primary inputs. The transducer $G$ has a transfer matrix

$$H = |G_1\, G_2\, G_3|. \tag{16}$$

Thus the output is related to the inputs by the equation

$$y_1 = G_1 z_1 + G_2 z_2 + G_3 z_3.$$

The desired system has a transfer matrix

$$H' = \left| \frac{G_1}{1 + G_4} G_2 G_3 \right|. \tag{17}$$

Thus the response of $y_1$ to the primary inputs is to be unchanged whereas the disturbance response is modified. The modification has been chosen completely arbitrarily in that $G_4$ may be any desired function.

Substitution of the expressions for $H$ and $H'$ into Eq. 12 yields the following matrix equation

$$\left| \frac{G_1}{1 + G_4} G_2 G_3 \right| = |G_1 G_2 G_3| \cdot \begin{vmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{vmatrix} + |G_1 G_2 G_3| \begin{vmatrix} k_{11} \\ k_{21} \\ k_{31} \end{vmatrix} \left| \frac{G_1}{1 + G_4} G_2 G_3 \right|.$$

This equation represent three simultaneous equations in twelve unknowns, the $c$'s and the $k$'s. As in the previous example, the fact that $z_1$ is a disturbance input and so cannot be modified enables certain constraints to be derived. These constraints are

$$\begin{aligned} c_{11} &= 1 \quad c_{21} = 0 \\ c_{12} &= 0 \quad c_{31} = 0 \\ c_{13} &= 0 \quad k_{11} = 0. \end{aligned}$$

The twelve unknowns have now been reduced to six. However, for a unique solution, three more constraints must be added. These added constraints are at this point completely arbitrary although in a particular problem they may be specified. The possibility of adding arbitrary constraints distinguishes this problem from the previous one and is a common feature of multi-variable control system design. Effectively it means that there may be an unlimited number of ways of synthesizing the desired system and one of the problems which arises out of this fact is that of selecting the optimum design from among the many possible designs. Two possible systems will now be derived for the problem of this section.

As a first approach, the following constraints may be added

$$\begin{aligned} c_{32} &= 0 \\ c_{23} &= 0 \\ k_{31} &= 0. \end{aligned} \tag{18}$$

This means that feedback will be applied to only one input and that there will be no cross-feeds between the input variables. The simultaneous equations for the system now become

$$\frac{G_1}{1+G_4} = G_1 + \frac{G_1 G_2}{1+G_4} k_{21} \tag{19}$$
$$G_2 = G_2 c_{22} + G_2^2 k_{21}$$
$$G_3 = G_3 c_{33} + G_3 G_3 k_{21}.$$

Solving these equations yields expressions for the three unknowns:

$$c_{22} = 1 + G_4$$
$$c_{33} = 1 + G_4 \tag{20}$$
$$k_{21} = \frac{-G_4}{G_2}.$$

Substituting Eqs. 19 into Eq. 4 gives

$$z_1 = x_1$$
$$z_2 = (1+G_4)\,x_2 - \frac{G_4}{G_2} y_1 \tag{21}$$
$$z_3 = (1+G_4)\,x_3,$$

where $x_1$, $x_2$ and $x_3$ are the final system inputs. The system represented by Eqs. 20 is shown in Fig. 7.

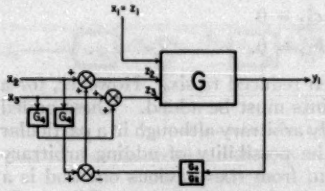


FIG. 7. First variation of three input–one output control system.

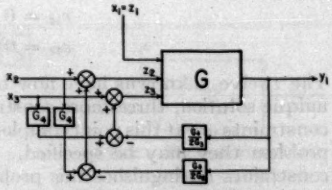


FIG. 8. Second variation of three input–one output control system.

An alternative solution of this design problem may be obtained by changing the constraints arbitrarily chosen in Eqs. 18. Let the following constraints be chosen:

$$c_{32} = 0$$
$$c_{23} = 0 \tag{22}$$
$$k_{21} = -\frac{G_4}{2G_2}.$$

In this case feedback will be applied to both input variables. Solution of the simultaneous equations now gives for the three unknowns:

$$\begin{aligned} c_{22} &= 1 + G_4 \\ c_{33} &= 1 + G_4 \\ k_{21} &= \frac{-G_4}{2G_2}. \end{aligned} \tag{23}$$

Substitution of Eqs. 23 into Eq. 4 gives:

$$\begin{aligned} z_1 &= x_1 \\ z_2 &= (1 + G_4)\, x_2 - \frac{G_4}{2G_2}\, y_1 \\ z_3 &= (1 + G_4)\, x_3 - \frac{G_4}{2G_3}\, y_1. \end{aligned} \tag{24}$$

The system represented by Eqs. 24 is shown in Fig. 8. The two systems shown in Figs. 7 and 8 are identical in their input-output response but have different configurations. These are, in fact, but two of an infinite number of possible systems which have the over-all transfer matrix given in Eq. 17.

#### 4. REALIZABILITY OF SYSTEMS

In the last section it was seen that the synthesis problem consisted of solving a set of $mn$ equations, represented by Eq. 12, for $n^2 + mn$ unknowns which are the elements of the matrices $C$ and $K$. It is not always possible to obtain a solution for such a set of equations and thus it follows that it is not always possible to design a computer with the form given in Eq. 4 which will enable a desired system matrix $H'$ to be obtained from the given system $H$. For this reason some method of determining whether the desired system is realizable is desired.

*First Realizability Criterion*

The $ij^{\text{th}}$ equation of the set of $mn$ simultaneous equations is given by Eq. 13

$$h'_{ij} = \sum_{q=1}^{n} h_{iq}\, c_{qj} + \sum_{l=1}^{m} \sum_{q=1}^{n} h_{iq}\, k_{ql}\, h'_{lj} \tag{13}$$

$$i = 1,2, \cdots m$$

$$j = 1,2, \cdots n.$$

To derive the first realizability criterion, it is necessary to express this set of equations in an alternative manner. The elements $h'_{ij}$ may be used to form a vector $B$ by taking them in the specific order $h'_{11}$,

$h'_{12}, \cdots h'_{1n}, h'_{21}, \cdots h'_{mn}$. Hence $B$ is an $mn \times 1$ column vector. Similarly, the unknown elements of $C$ and $K$ may be used to form a vector $Z$ by taking them in the specific order $c_{11}, c_{12}, \cdots c_{1n}, c_{21}, \cdots c_{nn}, k_{11}, k_{12}, \cdots k_{1m}, k_{21}, \cdots k_{mn}$. Hence $Z$ is an $(n^2 + mn) \times 1$ column vector.

The set of $mn$ equations may now be written in the following form:

$$AZ = B, \tag{25}$$

where $A$ is an $mn \times (n^2 + mn)$ matrix relating $B$ and $Z$. A further matrix $A^+$, the augmented matrix, is now formed by adding column $B$ to $A$ so as to form the $(n^2 + mn + 1)^{th}$ column of $A$. From the theory of simultaneous equations it now follows that the system of $mn$ equations may be solved for the elements of $C$ and $K$ if and only if

$$\text{rank } A = \text{rank } A^+. \tag{26}$$

$$\begin{array}{cccccccccccccccc}
c_{11} & c_{12} & \cdots & c_{1n} & c_{21} & c_{22} & \cdots & c_{2n} & c_{31} & c_{32} & \cdots & c_{3n} & \cdots & c_{n1} & c_{n2} & \cdots & c_{nn} \\
h_{11} & 0 & \cdots & 0 & h_{12} & 0 & \cdots & 0 & h_{13} & 0 & \cdots & 0 & \cdots & h_{1n} & 0 & \cdots & 0 \\
0 & h_{11} & \cdots & 0 & 0 & h_{12} & \cdots & 0 & 0 & h_{13} & \cdots & 0 & \cdots & 0 & h_{1n} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & h_{11} & 0 & 0 & \cdots & h_{12} & 0 & 0 & \cdots & h_{13} & \cdots & 0 & 0 & \cdots & h_{1n} \\
h_{21} & 0 & \cdots & 0 & h_{22} & 0 & \cdots & 0 & h_{23} & 0 & \cdots & 0 & \cdots & h_{2n} & 0 & \cdots & 0 \\
0 & h_{21} & \cdots & 0 & 0 & h_{22} & \cdots & 0 & 0 & h_{23} & \cdots & 0 & \cdots & 0 & h_{2n} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & h_{21} & 0 & 0 & \cdots & h_{22} & 0 & 0 & \cdots & h_{23} & \cdots & 0 & 0 & \cdots & h_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
h_{m1} & 0 & \cdots & 0 & h_{m2} & 0 & \cdots & 0 & h_{m3} & 0 & \cdots & 0 & \cdots & h_{mn} & 0 & \cdots & 0 \\
0 & h_{m1} & \cdots & 0 & 0 & h_{m2} & \cdots & 0 & 0 & h_{m3} & \cdots & 0 & \cdots & 0 & h_{mn} & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & h_{m1} & 0 & 0 & \cdots & h_{m2} & 0 & 0 & \cdots & h_{m3} & \cdots & 0 & 0 & \cdots & h_{mn}
\end{array}$$

$n$ rows; $mn$ rows

$\leftarrow n$ columns $\rightarrow$

$\leftarrow n^3$ columns $\rightarrow$

The rank of a matrix may be interpreted as the number of independent rows (or columns) of the matrix. Thus the realizability criterion reads:

A desired system is mathematically realizable if and only if the ranks of $A$ and $A^+$ are identical where $A$ and $A^+$ are as defined above for the system.

It should be reiterated here that physical realizability is not implied in this criterion, only the realizability of a solution for the elements of $C$ and $K$.

The determination of matrix $A$ becomes somewhat difficult for cases in which there are more than a small number of equations. It would be very desirable to be able to obtain $A$ without the necessity of writing out the system equations. That this is actually possible may be seen

by writing out the simultaneous equations explicitly as follows. From Eq. 13, the $ij^{th}$ equation is

$$h'_{ij} = h_{i1}\,c_{1j} + h_{i2}\,c_{2j} + h_{i3}\,c_{3j} + \cdots + h_{in}\,c_{nj} + \sum_{l=1}^{m} h'_{ij}\,(h_{i1}\,k_{1l} + h_{i2}\,k_{2l} + h_{i3}\,k_{3l} + \cdots + h_{in}\,k_{nl}).$$

Writing down the coefficients of the unknowns in the first part of the right hand sides of the $mn$ equations taken in the correct order the matrix shown on page 360 is obtained.

Similarly, by writing down the coefficients of the unknowns in the second part of the right hand side of the $mn$ equations taken in the correct order, the following matrix is obtained:

| $k_{11}$ | $k_{12}$ | .. | $k_{1n}$ | $k_{21}$ | $k_{22}$ | .. | $k_{2n}$ | $k_{31}$ | $k_{32}$ | .... | $k_{mn}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $h'_{11}h_{11}$ | $h'_{21}h_{11}$ | .. | $h'_{m1}h_{11}$ | $h'_{11}h_{12}$ | $h'_{21}h_{12}$ | .. | $h'_{m1}h_{12}$ | $h'_{11}h_{13}$ | $h'_{21}h_{13}$ | .... | $h'_{m1}h_{1n}$ |
| $h'_{12}h_{11}$ | $h'_{22}h_{11}$ | .. | $h'_{m2}h_{11}$ | $h'_{12}h_{12}$ | $h'_{22}h_{12}$ | .. | $h'_{m2}h_{12}$ | $h'_{12}h_{13}$ | $h'_{22}h_{13}$ | .... | $h'_{m2}h_{1n}$ |
| ..... | ..... | | ..... | ..... | ..... | | ..... | ..... | ..... | .... | ..... |
| $h'_{1n}h_{11}$ | $h'_{2n}h_{11}$ | .. | $h'_{mn}h_{11}$ | $h'_{1n}h_{12}$ | $h'_{2n}h_{12}$ | .. | $h'_{mn}h_{12}$ | $h'_{1n}h_{13}$ | $h'_{2n}h_{13}$ | .... | $h'_{mn}h_{1n}$ |
| $h'_{11}h_{21}$ | $h'_{21}h_{21}$ | .. | $h'_{m1}h_{21}$ | $h'_{11}h_{22}$ | $h'_{21}h_{22}$ | .. | $h'_{m1}h_{22}$ | $h'_{11}h_{23}$ | $h'_{21}h_{23}$ | .... | $h'_{m1}h_{2n}$ |
| $h'_{12}h_{21}$ | $h'_{22}h_{21}$ | .. | $h'_{m2}h_{21}$ | $h'_{12}h_{22}$ | $h'_{22}h_{22}$ | .. | $h'_{m2}h_{22}$ | $h'_{12}h_{23}$ | $h'_{22}h_{23}$ | .... | $h'_{m2}h_{2n}$ |
| ..... | ..... | | ..... | ..... | ..... | | ..... | ..... | ..... | .... | ..... |
| $h'_{1n}h_{21}$ | $h'_{2n}h_{21}$ | .. | $h'_{mn}h_{21}$ | $h'_{1n}h_{22}$ | $h'_{2n}h_{22}$ | .. | $h'_{mn}h_{22}$ | $h'_{1n}h_{23}$ | $h'_{2n}h_{23}$ | .... | $h'_{mn}h_{2n}$ |
| ..... | ..... | | ..... | ..... | ..... | | ..... | ..... | ..... | .... | ..... |
| $h'_{11}h_{m1}$ | $h'_{21}h_{m1}$ | .. | $h'_{m1}h_{m1}$ | $h'_{11}h_{m2}$ | $h'_{21}h_{m2}$ | .. | $h'_{m1}h_{m2}$ | $h'_{11}h_{m3}$ | $h'_{21}h_{m3}$ | .... | $h'_{m1}h_{mn}$ |
| $h'_{12}h_{m1}$ | $h'_{22}h_{m1}$ | .. | $h'_{m2}h_{m1}$ | $h'_{12}h_{m2}$ | $h'_{22}h_{m2}$ | .. | $h'_{m2}h_{m2}$ | $h'_{12}h_{m3}$ | $h'_{22}h_{m3}$ | .... | $h'_{m2}h_{mn}$ |
| ..... | ..... | | ..... | ..... | ..... | | ..... | ..... | ..... | .... | ..... |
| $h'_{1n}h_{m1}$ | $h'_{2n}h_{m1}$ | .. | $h'_{mn}h_{m1}$ | $h'_{1n}h_{m2}$ | $h'_{2n}h_{m2}$ | .. | $h'_{mn}h_{m2}$ | $h'_{1n}h_{m3}$ | $h'_{2n}h_{m3}$ | .... | $h'_{mn}h_{mn}$ |

$n$ rows; $mn$ rows

$m$ columns; $mn$ columns

Examination of these two matrices shows that the first may be written more briefly as:

$$\begin{vmatrix} Ih_{11} & Ih_{12} \cdots Ih_{1n} \\ Ih_{21} & Ih_{22} \cdots Ih_{2n} \\ \cdots\cdots\cdots\cdots \\ Ih_{m1} & Ih_{m2} \cdots Ih_{mn} \end{vmatrix}$$

where $I$ is the $n \times n$ identity matrix.

The second matrix may be written more briefly as:

$$\begin{vmatrix} H'^{*}h_{11} & H'^{*}h_{12} \cdots H'^{*}h_{1n} \\ H'^{*}h_{21} & H'^{*}h_{22} \cdots H'^{*}h_{2n} \\ \cdots\cdots\cdots\cdots \\ H'^{*}h_{m1} & H'^{*}h_{m2} \cdots H'^{*}h_{mn} \end{vmatrix},$$

where $H'^*$ denotes the transpose of $H'$. Hence the matrix $A$ may be written:

$$A = \begin{vmatrix} Ih_{11}\ Ih_{12}\cdots Ih_{1n}\ H'^*h_{11}\ H'^*h_{12}\cdots H'^*h_{1n} \\ Ih_{21}\ Ih_{22}\cdots Ih_{2n}\ H'^*h_{21}\ H'^*h_{22}\cdots H'^*h_{2n} \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ Ih_{m1}\ Ih_{m2}\cdots Ih_{mn}\ H'^*h_{m1}\ H'^*h_{m2}\cdots H'^*h_{mn} \end{vmatrix}.$$

Thus $A$ may be written directly from a knowledge of $H$ and $H'$ without the necessity of writing out the simultaneous equations describing the system.

*Second Realizability Criterion*

A criterion for realizability of any desired system will now be derived in which it is shown that realizability is dependent upon the relationship between the rows of $H$ and between the rows of $H'$. Consider first the case in which the rows of $H$ are all independent. That is, no row may be made up of a linear combination of other rows in the matrix, treating the rows as vectors. It will be of interest to determine the rank of $A$ in this case.

By examination of the arrays of coefficients comprising $A$, which were shown previously, it may be seen that the group of $n$ rows beginning with $I\,h_{11}$, in Eq. 28, are mutually independent. That is, no row can be made up of a linear combination of any of the other rows. Similarly the next group of $n$ rows and all succeeding groups of $n$ rows are each mutually independent. However, since the rows of $H$ are all independent, it follows that all of the rows of $A$ are independent. Thus the rank of $A$ is $mn$. (The rank of a non-square matrix $\leqslant$ minimum dimension of the matrix.) It is obvious that the addition of column $B$ to $A$ will not change this situation. Therefore the rank of $A^+$ is also $mn$. This result has been obtained without consideration of $H'$. Thus it follows that, in this case, rank $A$ = rank $A^+$ for any $H'$. Hence any desired system $H'$ is realizable when the rows of $H$ are independent.

Consider now the case in which the rows of $H$ and of $H'$ are not all independent and let the relationship among the rows be identical for both $H$ and $H'$. Referring to the arrays of coefficients given earlier, it is apparent that, as before, each group of $n$ rows of $A$ is mutually independent. It is now necessary to consider the effect on $A$ of a linear relationship between the rows of $H$. As a simple example, first of all, let the $m^{th}$ row of $H$ be equal to the sum of the first and second rows of $H$. Hence it follows that:

$$\begin{aligned} h_{11} + h_{21} &= h_{m1} \\ h_{12} + h_{22} &= h_{m2} \\ &\cdots\cdots\cdots \\ h_{1n} + h_{2n} &= h_{mn} \end{aligned} \qquad (29)$$

Consider now the sum of the 1st and the $(n+1)^{st}$ rows of $A$. It may be seen that for each column the sum of the elements in the first row and the $(n+1)^{st}$ row is equal to the corresponding element of the $(mn-n+1)^{st}$ row. It also may be seen that the second row plus the $(n+2)^{nd}$ row equals the $(mn-n+2)^{nd}$ row, the third row plus the $(n+3)^{rd}$ row equals the $(mn-n+3)^{rd}$ row and so on for all of the first, second and $m^{th}$ groups of $n$ rows.

In general, if a linear relationship exists among the rows of $H$, non-zero coefficients $a_k$ exist such that

$$\sum_{k=1}^{m} a_k E_k = 0, \tag{30}$$

where the $E$'s are the rows of $H$. As illustrated by the simple example just given, it may be seen that a corresponding set of relationships occurs among the rows of $A$:

$$\begin{aligned}
&\sum_{k=1}^{m} a_k F_{nk-n+1} = 0 \\
&\sum_{k=1}^{m} a_k F_{nk-n+2} = 0 \\
&\cdots\cdots\cdots\cdots\cdots \\
&\sum_{k=1}^{m} a_k F_{nk} = 0,
\end{aligned} \tag{31}$$

where the $F$'s are the rows of $A$ and the $a$'s are the same as those in Eq. 30. It has been seen that the rows of $A$ contained in each of the $n$ summations are independent of the rows in any other summation. Therefore it may be concluded that if there are $q$ independent rows in $H$ there must be $nq$ independent rows in $A$. In other words, the rank of $A$ is $nq$.

There are also $q$ independent rows in $H'$ and this matrix has the same relationship among its rows as that of $H$. Consider the effect of adding column $B$ to matrix $A$. The element $h'_{11}$ will be added to the first row of $A$, $h'_{21}$ to the $(n+1)^{th}$ row, $h'_{31}$ to the $(2n+1)^{th}$, $\cdots h'_{m1}$ to the $(mn-n+1)^{th}$ row. Element $h'_{12}$ will be added to the second row of $A$, $h'_{22}$ to the $(n+2)^{nd}$ row, $h'_{32}$ to the $(2n+2)^{nd}$, and so on.

Since the rows of $H'$ are related in the same way as those of $H$, a set of relationships may be given among the rows of $B$:

$$\begin{aligned}
&\sum_{k=1}^{m} a_k h'_{k1} = 0 \\
&\sum_{k=1}^{m} a_k h'_{k2} = 0 \\
&\cdots\cdots\cdots\cdots\cdots \\
&\sum_{k=1}^{m} a_k h'_{kn} = 0
\end{aligned} \tag{32}$$

where the $a$'s are the same as those in Eq. 31. It may be seen that the row relationship of $B$ is the same as that of $A$ and hence the number of independent rows in $A^+$ is equal to the number of independent rows in $A$. This shows that the ranks of $A$ and $A^+$ are both equal to $nq$. Thus it may be concluded that $H'$ is realizable when its rows are interrelated in the same manner as those of $H$.

It now remains to show that the existence of dissimilar row relationships in $H$ and $H'$ always implies that rank $A \neq$ rank $A^+$. A new set of relationships among the rows of $B$ may be written when $H'$ has a different row relationship.

$$\sum_{k=1}^{m} b_k h'_{k1} = 0$$

$$\sum_{k=1}^{m} b_k h'_{k2} = 0 \qquad (33)$$

$$\cdots\cdots\cdots\cdots$$

$$\sum_{k=1}^{m} b_k h'_{kn} = 0$$

where the $b$'s are not all identical to the corresponding $a$'s of Eq. 32. It therefore follows that the rows of $A^+$ are no longer related in the same way as those of $A$ and hence rank $A \neq$ rank $A^+$.

The second realizability criterion can now be stated as follows:

If the rows of $H$ are all independent, any system $H'$ is realizable. If the rows of $H$ are not all independent, $H'$ is realizable if and only if it has the same inter-row relationship as $H$.

From matrix theory it can be shown that this realizability criterion may be alternatively stated as follows:

A system is realizable if and only if $H'$ is equivalent to $H$ under elementary column operations alone.

This means that it must be possible to transform $H$ to $H'$ by means of elementary column operations. These operations are:

(*a*) multiplication of a column by any quantity including zero.
(*b*) addition of any multiple of any column to any other column.
(*c*) interchange of any two columns.

It should be noted that the second realizability criterion has been derived tacitly assuming that $m < n$. In the unlikely case where $m > n$, it would be necessary to consider inter-column relationships and the final statement of the criterion would involve elementary row operations instead of elementary column operations.

### 5. APPLICATION OF REALIZABILITY CRITERIA

Application of the two criteria derived in the previous section would be perfectly straightforward if the elements of the matrices were pure numbers. However, in any practical problem the matrix elements will normally be functions of the complex frequency variable $s$. It then becomes a more difficult matter to determine the rank and row relationship of a given matrix. A method of treating this problem is as follows.

If any matrix, whose elements are functions of $s$, has a certain rank or inter-row relationship, then it is apparent that the matrices obtained by substitution of any number, real, complex or imaginary, for $s$ will still have the same rank or inter-row relationship. Thus, if in a matrix, the third row is equal to the sum of the first and second rows, then this relationship will be true for any value of $s$.

Now consider the converse of this statement. If after substitution of a numerical value for $s$ in a matrix it is found that the third row is equal to the sum of the first and second rows, there are two possible conclusions which may be drawn. The first conclusion is that the third row is equal to the sum of the first and second rows for all values of $s$. The second possibility is that by chance, this particular value of $s$, after substitution in the elements of the first and second rows and addition of the corresponding numbers, happens to give the corresponding numbers obtained by substitution of this value of $s$ in the third row. This means that this particular value of $s$ happens to be a solution of the equation obtained for each column of the matrix by adding the elements in the first and second rows and equating the result to the element in the third row. It is evident that the probability of picking a value of $s$ such that this result occurs is very small. Obviously the probability of this result occurring for two different values of $s$ is extremely small. Hence a second substitution will remove almost all doubt concerning the interpretation of the result of the substitution. Thus the rank and inter-row relationship are obtained by substituting a numerical value for $s$ and examining the numerical matrix so obtained. For ease of calculation, it is apparent that integral real numbers should be substituted for $s$. Also, values of $s$ which are the poles of any of the elements of the matrix should be avoided in order that no infinite elements are obtained.

An example will now be given which illustrates this method and the use of the two realizability criteria.

A system is given whose transfer matrix is

$$H = \begin{vmatrix} \dfrac{1}{1+s} & \dfrac{4}{1+2s} \\ \dfrac{1}{1+3s} & \dfrac{1}{1+2s} \end{vmatrix}.$$

It is desired to diagonalize this matrix and obtain the following system

$$H' = \begin{vmatrix} \dfrac{1}{1+s} & 0 \\ 0 & \dfrac{1}{1+2s} \end{vmatrix}.$$

To determine whether this desired system is actually realizable, the first realizability criterion may be used. By use of Eq. 28, matrix $A$ is obtained as follows:

$$A = \begin{vmatrix} \frac{1}{1+s} & 0 & \frac{4}{1+2s} & 0 & \frac{1}{(1+s)(1+s)} & 0 & \frac{4}{(1+2s)(1+s)} & 0 & \frac{1}{1+s} \\ 0 & \frac{1}{1+s} & 0 & \frac{4}{1+2s} & 0 & \frac{1}{(1+s)(1+2s)} & 0 & \frac{4}{(1+2s)(1+2s)} & 0 \\ \frac{1}{1+3s} & 0 & \frac{1}{1+2s} & 0 & \frac{1}{(1+3s)(1+s)} & 0 & \frac{1}{(1+2s)(1+s)} & 0 & 0 \\ 0 & \frac{1}{1+3s} & 0 & \frac{1}{1+2s} & 0 & \frac{1}{(1+3s)(1+s)} & 0 & \frac{1}{(1+2s)(1+2s)} & \frac{1}{1+2s} \end{vmatrix}.$$

Column $B$ has been added to the right of $A$. To determine the ranks of $A$ and $A^+$, substitute $s = 0$ in the above matrix. The following matrix is obtained:

$$\left|\begin{array}{cccccccc|c} 1 & 0 & 4 & 0 & 1 & 0 & 4 & 0 & 1 \\ 0 & 1 & 0 & 4 & 0 & 1 & 0 & 4 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \end{array}\right|.$$

Examination of the columns of this matrix shows that

$$\text{rank } A = \text{rank } A^+ = 4.$$

Thus the desired system is realizable.

The second realizability criterion may now be used to check this result. Substitution of $s = 0$ in $H$ and $H'$ gives:

$$H = \begin{vmatrix} 1 & 4 \\ 1 & 1 \end{vmatrix} \quad H' = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}.$$

It may be seen that both of these matrices are non-singular. Hence they are equivalent under column operations and the desired system is realizable. It should be noted that a second substitution for $s$ was not necessary here since this procedure is only necessary when a definite relationship between the rows occurs after the first substitution.

*Acknowledgments*

The work described in this paper has been supported by awards from the National Research Council of Canada. The author is grateful to the above body for permission to publish the results of this work. The author also wishes to acknowledge the guidance and assistance of his Ph.D. supervisor, Dr. J. M. Ham of the Electrical Engineering Department at the University of Toronto.

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**Aluminum Alloy Laboratory Clamps.**—Many laboratory accidents can be prevented by using aluminum alloy clamps to hold glassware. Aluminum's high melting point provides far greater safety in the heating of inflammable or dangerous liquids, according to Stanley Dodd, technical director of Chicago Apparatus Co. He explained there is very little chance that an Alumaloy clamp will soften, dropping the glassware that contains the liquid.

Aluminum provides a safety factor of 75 per cent. Its melting point of 1220° F. compares with 700° F. for cast zinc.

Dodd said tests also show aluminum clamps have a tensile strength of 45,000 psi. In addition, aluminum has a natural resistance to corrosion, giving clamps a longer, more useful life, he pointed out.

Alumaloy clamps are available nationally. Price is competitive with cast zinc or cast iron clamps.

**Materials Tested by Bombardment.**—A noiseless bombardment under 20 ft. of water at the Hanford atomic plant is showing the way to better materials for atomic reactors and reactors capable of improved performance over a longer time and at high power levels.

Scene of this silent siege on various sample materials is the bottom of a storage basin pool at the rear face of a nuclear reactor, a veritable underwater no-man's land of atomic radiation. Targets used are rubber, plastics, metals and various other materials employed in reactor construction.

Waste energy from intensely radioactive fuel elements which have been discharged from the reactors and are being cooled in the deep pool fires the high energy gamma rays at the test targets.

Until the test program was instituted, the fuel elements were cooled in the same way, but the rays from the irradiated uranium were harmlessly and uselessly dissipated under 20 feet of water that fills the storage basins.

To perform the materials testing, General Electric operators remotely load the fuel elements into tubes welded to the sides of a cylinder-shaped apparatus devised for directing gamma rays at test materials. Samples to be bombarded are lowered into the cylinder until they lie in a direct line with the radiation from the uranium "slugs."

Some of the samples under test have been exposed to gamma bombardment for more than a year; others for only a few seconds. Nearly all show signs of radiation exposure.

Many rubber samples are reduced to shapeless globs. Polyethylene, the pliable plastic used in many squeeze bottles and moisture-proof vegetable bags, first turns brittle, then rubbery and finally cracks. Many types of glass take on a smoky brown hue after exposure to gamma radiation.

Data gained from these tests by General Electric research teams at the Atomic Energy Commission plant are helping determine effects of high-energy bombardment on reactor materials and components. These studies also are aiding in development of new materials for the future through rearrangement of molecules in existing materials.

Some materials tested already have been replaced in atomic reactors by others found more resistant to atomic radiation. Thus, better reactor designs are made possible.

# RESEARCHES ON CORROSION AND INHIBITION

## REACTION VELOCITY IN THE SYSTEM IRON: DILUTE ACETIC ACID AT 40° C.

BY

GEORGE S. GARDNER,[1] HARRY L. FAIGEN,[1] GREGORY L. GIBSON[1] AND WILBUR S. HALL[1]

Part I[2]

ABSTRACT

The rate-determining factors in corrosion reactions have been reviewed in the light of the work of Nernst, Langmuir, and others, and the characteristics of diffusion and activation control in such reactions have been discussed. Following Freundlich and Langmuir, corrosion inhibitors are assumed to act by adsorption on the metal surface. Rate equations containing the function $s$, the fractional surface covered by adsorbed inhibitor, are given to express the reaction velocity in diffusion and activation controlled systems.

A detailed discussion is given of equipment and procedure devised for studying the velocity of corrosion reactions under conditions where the amounts of corrodents, both liquid and gaseous, are under careful control.

A study of the corrosion system iron:acetic acid is made at 40° C., where the amount of acetic acid is 1000 ppm., in the absence of oxygen. The behavior of the system is studied in the presence and absence of corrosion inhibitors. Two commercial corrosion inhibitors were selected for study.

The results of the experiments have been plotted according to the velocity equation given for diffusion-controlled reactions and are shown to be in agreement with that equation. They can, however, agree with an activation-controlled mechanism. Further experiments at higher temperatures, now being carried out in this laboratory, should shed light on the actual mechanism.

The apparent value of $s$, the fractional surface covered by inhibitor, is calculated for each experiment. It is shown that $s$ is a useful concept in corrosion testing, and that inhibitors can be evaluated and their inhibitor efficiency expressed in terms of this quantity.

In the experimental work, two series of corrosion experiments were carried out: one in the presence of an oil phase (mineral spirits), and one in the absence of an oil phase. The experiments showed a very considerable acceleration of the reaction velocity in the presence of the oil phase. The possible influence of an oil phase in corrosion reactions is discussed.

Of the two inhibitors studied, one exerts its inhibiting action either in the presence or absence of an oil phase. Apparently it is strongly adsorbed. The other inhibitor shows some inhibiting action in the presence of an oil phase, but little in the absence of the oil phase. This casts some interesting light on the "adsorption" and "water displacing" theories of inhibitor action in oil wells.

Corrosion reactions are reactions between solid, liquid, and gaseous phases. Heterogeneous reactions of this nature have been studied by

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[2] Part II will appear in this JOURNAL for December, 1956.

many investigators (1, 2, 3, 4).[3] Corrosion reactions are frequently complex, and are further complicated by the fact that the products of reaction can be soluble or insoluble in the liquid phase. If the reaction products are soluble or dispersible in the liquid phase, as in the solution of iron in acids, the problems of reaction velocities in such systems can be attacked by conventional methods. If, however, the reaction products are solid, and are not soluble or dispersible in the liquid phase, and tend to form increasingly thick coatings of solid reaction products on the surface of the solid reactant, the methods of studying reaction velocities in such systems might differ widely from conventional procedures. Although studies on the oxidation of metals have succeeded to some extent (3) in systematizing the study of this portion of the field, other reactions between solids and gases where the reaction product is another solid have shown that such reactions can be of unusual complexity, and require unusual methods for their study (4).

In this investigation it is reactions of the first type that interest us—reactions in which the products of the reaction are soluble or dispersible in the fluid medium. Particularly, it is our object to study the reaction between steel and dilute acetic acid, under such conditions as are related to the corrosion of steel in sub-surface oil-well equipment.

### RATE DETERMINING STEPS IN CORROSION REACTIONS

It seems reasonable to believe, following most modern investigators in the field of heterogeneous reactions, that reactions between solids and liquids occur in the following stages (11): transport of the reactant to the surface; adsorption of the reactant; reaction with, or on, the surface; desorption of the products of reaction; and transport of the liberated products from the surface into the bulk phase.

It is evident that these stages can apply to the reaction of liquids *with* solids, as in the corrosion reactions, as well as to the reaction of liquids and gases *at the surface of* solids, as in catalytic reactions. It is also evident that the slowest of the processes will determine the over-all reaction rate.

In the study of corrosion reactions we need not distinguish between adsorption and reaction at the surface, and will divide the above reaction stages into two general types: transport stages and activation stages.

If, therefore, the transport stages are slow compared to the activation stages, the over-all velocity will be controlled by the rate at which reactant reaches and leaves the reacting surface. On the other hand, if the activation energy of the reaction is very high, the actual reaction at the surface might be the slow process, and might control the reaction velocity.

[3] The boldface numbers in parentheses refer to the references appended to Part I of the paper.

### CHARACTERISTICS OF DIFFUSION-CONTROLLED REACTIONS (15)

If transport stages in the reaction are relatively slower than activation stages, we say that the reaction is diffusion-controlled. The following criteria are usually set up for the characterization of diffusion-controlled reactions:

The activation energy of the surface reaction is relatively low.

The velocity of agitation influences the observed over-all rate of reaction.

The observed reaction velocity should follow the equation for a first-order process, for diffusion is a first-order process.

The observed reaction velocity is a function of the diffusion coefficient $D$ and the thickness of the diffusion layer $h$ (see below). This is equivalent to saying that the observed reaction velocity is independent of the nature of the solid. (This is not generally recognized.)

The final criterion is the temperature coefficient. The rate of diffusion is proportional to the square root of the temperature.

### CHARACTERISTICS OF ACTIVATION-CONTROLLED REACTIONS (15)

Activation-controlled reactions usually conform to the following:

The orders of surface reactions that are controlled by the activation energy can be 0, 1, 2, or any value between 0 and 2 (14).

The rate varies exponentially with the temperature. The old rule is that velocities are about doubled for each 10° C. rise in temperature.

Velocities are unaffected by changes in the rate of agitation.

The rate of the actual surface reaction should be very much slower than the diffusion rate.

A change in the type of the solid, other conditions being held the same, can profoundly affect the reaction velocity.

### THE NERNST-BRUNNER EQUATION FOR THE VELOCITY OF DIFFUSION-CONTROLLED REACTIONS

When a solid and a fluid medium react, it was recognized quite early that the rate of transfer of reactants and products to and from the phase boundary is frequently the controlling factor in the over-all reaction velocity. Nernst and Brunner (7), building on the work of Noyes and Whitney (6) and Fick (5), proposed an equation for the expression of the reaction velocity between solid and liquid phases, and used this equation as the basis for their diffusion theory of reaction velocity in heterogeneous systems. (It is hardly necessary to point out here that we no longer consider the Nernst diffusion theory valid for catalytic reactions.)

The Nernst-Brunner equation is usually expressed as

$$-\frac{dm}{dt} = \frac{A}{h}\frac{D}{V}m \qquad (1)$$

where $m$ is the number of gram-moles of reactant present at time $t$ in the solution, $A$ is the original area of the reacting surface (the phase boundary), $h$ is the thickness of the Nernst "film," $D$ is the diffusion coefficient, and $V$ is the volume of the solution.

As an inherent part of this equation Nernst assumed:

(*a*) There is a definite "diffusion layer," through which it is necessary for a solute to diffuse before reacting with the solid surface. He actually gave calculated measurements of the thickness of this "film."
(*b*) The actual reaction at the phase boundary is extremely rapid.
(*c*) Reactants and products diffuse slowly through the "film."

Many investigators have contributed to and modified the Nernst theory (8, 9, 10), and today we would reword Nernst's postulates as follows:

(*a*) The revised picture of the Nernst "film" is of a layer next to the solid surface, still in "turbulent" flow, but in which the velocity component of turbulence normal to the interface is a negligible factor in transporting reactant, compared to diffusion (9).
(*b*) For a diffusion-controlled mechanism, the actual reaction at the phase boundary need not be extremely rapid. It must, however, be appreciably faster than the rate of diffusion.

### AN EXPRESSION FOR THE REACTION VELOCITY OF ACTIVATION-CONTROLLED REACTIONS

Using the same nomenclature as above, we can express the velocity of activation-controlled reactions as

$$-\frac{dm}{dt} = k A \left(\frac{m}{V}\right)^n, \qquad (2)$$

where $k$ is the true reaction velocity constant, and $n$ varies between 0 and 2.

### INHIBITORS OF REACTION IN HETEROGENEOUS SYSTEMS

The work of Freundlich (17), Langmuir (2, 14), and many others, on the mechanics of adsorption on solid surfaces, have given us a logical basis on which to build a theory of inhibition in corrosion systems. Following these investigators, we assume that organic inhibitors of corrosion are adsorbed on metal surfaces and exert their inhibiting action through a blanketing effect on the active metal surface (12). The amount of reaction taking place is then a function of the uncovered surface. The expression of the fractional surface covered by adsorbed inhibitor is:

*According to Freundlich:*

$$s = b\,c^{1/n}. \tag{3}$$

*According to Langmuir:*

$$s = \frac{a\,c}{1 + b\,c}. \tag{4}$$

In these equations $s$ is the fraction of the solid surface covered by adsorbed inhibitor, $c$ is the molar concentration of adsorbed inhibitor, and $a$, $b$ and $n$ are constants.

Equation 1 can then be rewritten as

$$-\frac{dm}{dt} = \frac{(1 - s)\,A\,D}{h}\frac{m}{V}, \tag{5}$$

where $(1 - s)\,A$ is now the surface area which enters into reaction, and Eq. 2 can be rewritten as

$$-\frac{dm}{dt} = k\,(1 - s)\,A\left(\frac{m}{V}\right)^{n}. \tag{6}$$

### AQUEOUS AND NON-AQUEOUS PHASES, AND THEIR POSSIBLE EFFECT ON THE REACTION VELOCITY OF CORROSION

It is interesting to speculate on the behavior of aqueous and non-aqueous (oily) phases in corrosion systems, and their effect on the reaction velocity. The following points are presented for consideration:

1. We would expect that the rate of diffusion of a reactant through a Nernst "film" would be markedly affected by the type of fluid of which the "film" is composed. If, for example, the "film" were oily, the rate of diffusion of a water-soluble reactant would certainly be extremely slow.
2. In a rapidly mixing system of water and oil, the Nernst "film" might tend to be a dispersion of water and oil. The thickness could vary considerably from the thickness of a water film.
3. In a rapidly mixing system of water and oil, under certain conditions, water-displacing agents are effective in slowing down the rate of corrosion. Under these conditions oil adsorption can be increased. These agents might also increase the ratio of oil in the Nernst "film."
4. There is, on the other hand, the possibility that a *discontinuous* oil film might accelerate corrosion by causing more marked differences between cathodic and anodic areas.
5. In addition to the above, there is the matter of hydrogen polarization. An oily film would probably affect the rate of hydrogen removal. It seems difficult to predict what this effect might be.

### EXPERIMENTAL METHODS AVAILABLE FOR THE MEASUREMENT OF REACTION VELOCITY IN CORROSION SYSTEMS

Information on the measurement of reaction velocity is available in many places. One of the very thorough discussions is that of Friess and Weissberger (18). Since it is our object to measure corrosion velocities in the iron:acetic acid system, this discussion is directed particularly to that system.

If iron is being dissolved by dilute acetic acid (in the presence or absence of inhibitors) the data required for expression of the reaction velocity of the solution process are: numerical data on the disappearance of one of the reactants (solid or solute); or numerical data on the appearance of a product.

We can, of course, determine the course of the disappearance of iron with time, by direct weighing, for example. We can also determine the amount of acid present at various values of the time by one of the conventional procedures.

As to the measurement of the amount of a product formed at a given time, the one most frequently used is the measurement of the hydrogen formed. The measurement of iron in solution is another possibility.

In most cases, however, and particularly where extremely dilute acids are under consideration, the most convenient, and usually the best, method, is the first mentioned: the determination of the amount of iron dissolved at a given time by direct weighing.

### QUANTITATIVE METHODS OF MEASUREMENT USED IN CORROSION TESTING, AND THEIR APPLICATION TO THE MEASUREMENT OF REACTION VELOCITIES IN CORROSION SYSTEMS

Many of the procedures used in corrosion testing have possibilities for the measurement of reaction velocities in corrosion systems. On the other hand, many test methods, while giving useful data, showing satisfactory correlation with field tests, have no direct applicability to the measurement of the velocity of corrosion processes. It is very difficult, for example, to apply measurements of electrode potential (19, 20, 21), polarization (19), electrical film resistance (19, 22) and surface and interfacial tension effects (19) to the measurement of velocity in the particular system under study, although some of these measurements give information of definite value. However, corrosion tests where evolved hydrogen (13), dissolved iron (19), or loss in weight of a test specimen (19) are measured, are always a possibility for reaction velocity measurements, providing that the accompanying experimental conditions are satisfactory.

### REQUIRED CONDITIONS FOR REACTION VELOCITY MEASUREMENTS

In starting our experimental work on the measurement of the reaction velocity of the iron:dilute acetic acid reaction, we set up the following requirements that must be met by our experimental systems: con-

stant temperature, within a reasonable limit; constant rate of stirring or mixing (which could be varied at will); uniformity of the metal test specimens; and a procedure for introducing exact quantities of gases (inert gases or reactants) into the reaction system.

## EQUIPMENT AND MATERIALS USED FOR THE MEASUREMENT OF REACTION VELOCITY IN THE IRON:ACETIC ACID SYSTEM

### *Materials*

*Nitrogen* used was Matheson prepurified Nitrogen, from the No. 2 cylinder.

*Brine* was sodium chloride brine, 40 g. NaCl (reagent grade) per liter of solution.

*Acetic acid* was Merck Reagent glacial acetic acid.

*Oil phase*, when one was used in the experiments, was mineral spirits, thoroughly washed with distilled water. (Amsco 460 solvent.)

*Steel* used for the specimens was a hot-rolled SAE 1010 strip, obtained from a single heat in a quantity of half a ton, and used in our laboratories for all our standardized acid corrosion tests, where a mild steel is needed. This standard steel was prepared by the following procedure:

> The first operation is to cut a considerable portion of the steel into strips 7 in. in length. These strips are wiped carefully with isopropyl alcohol to remove any oily material. Any badly corroded pieces are discarded and any pieces that have suffered unusual distortion in the rolling are discarded. The strips are then immersed in 1:1 HCl at 25° C. for 10 min. They are then removed and immediately immersed completely in water. The strips are removed from the water in pairs, washed thoroughly under running hot water, dried quickly with clean cheesecloth, and placed in an oven at 110° C. for 20 min. The strips are then removed and placed in a clean desiccator charged with fresh silica gel. They are weighed as needed.

*The inhibitors* used were two commercial materials that are used in sweet well treatment. They are designated as *A* and *B*.

### *The Reaction Vessel*

The reaction vessel is shown in Fig. 1. It is a bulb connected by means of a standard taper ground joint with a standard taper stopcock. The approximate dimensions are: diameter of bulb, 34 mm.; length of bulb, 150 mm.; and over-all length, 325 mm.

### *Apparatus Used for Charging the Reaction Vessel under Gas-Tight Conditions*

Figure 2 shows the system used for manipulating the reaction vessel under gas-tight conditions. The system is essentially a manifold con-

nected to a manometer, the manifold containing seven stopcocks, which are used for various types of corrosion studies, as shown in Fig. 3.

Rubber tubing is used for making the various connections, the most satisfactory tubing being AHT nitrometer tubing, No. 8842, 3/16 in.

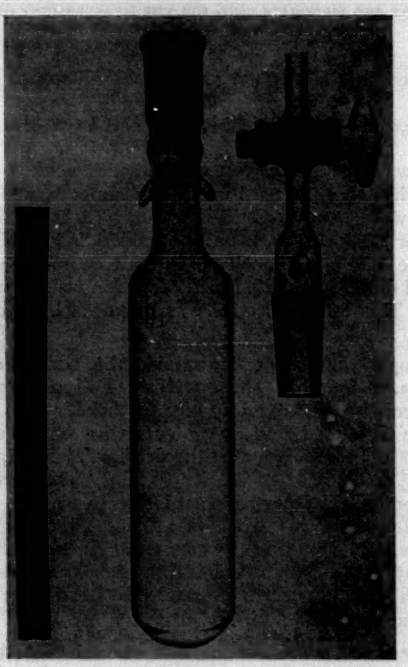

FIG. 1.

bore, 1/8 in. wall, for all connections except No. 2 connection (reaction vessel), and for this purpose AHT 8834, 1/4 in. bore, 5/64-in. wall is used.

The glass portions of the apparatus are of Pyrex, and should be well made and well annealed to withstand a reasonable amount of usage. For the manifold Dow-Silicone stopcock grease is used, except that for the reaction vessel and its connections the special lubricant described below is used.

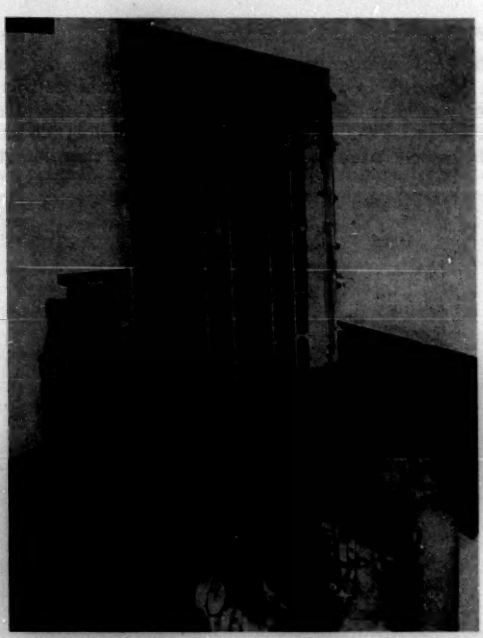

FIG. 2.

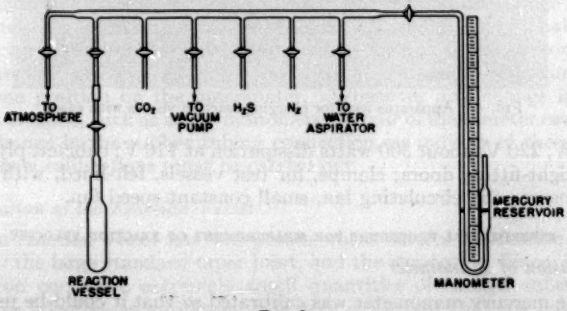
TO ATMOSPHERE
$CO_2$
TO VACUUM PUMP
$H_2S$
$N_2$
TO WATER ASPIRATOR
MERCURY RESERVOIR
REACTION VESSEL
MANOMETER

FIG. 3.

*Mixing Equipment*

The constant temperature cabinet containing the tube-turning equipment is shown in Fig. 4. Although the cabinet was developed independently of that described by Wachter (16), the similarity is evident. The essential elements of the installation are: motor and drive, with variable speed reducer; thermoregulator and thermometer; heater,

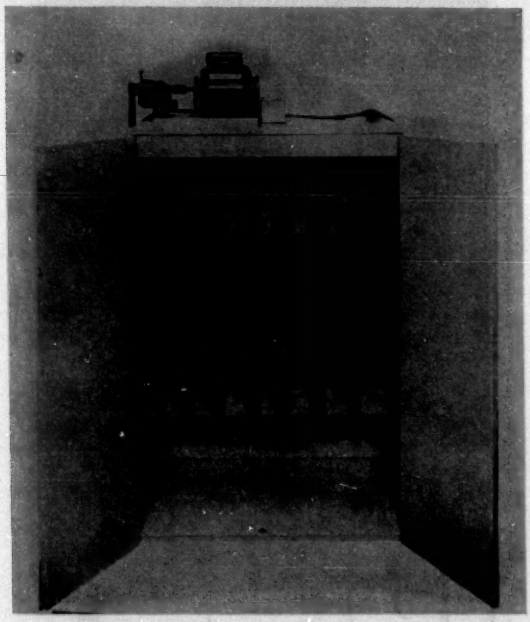

FIG. 4. Apparatus used for charging reaction vessels with gases.

1500 W, 220 V, about 500 watts dissipation at 110 V; cabinet, plywood, with tight-fitting doors; clamps, for test vessels, felt-lined, with wing-nut closure; and circulating fan, small constant speed fan.

## EXPERIMENTAL PROCEDURE FOR MEASUREMENT OF REACTION VELOCITY

*Calibration of Manometer*

The mercury manometer was calibrated so that it could be used for measuring the quantity of all gases admitted to the reaction vessel. Since the pressures were sufficiently low that the deviation from a per-

fect gas was negligible under the experimental conditions, it was convenient to make all measurements by referring them to the equivalent volume of hydrogen sulfide measured by the manometer.

The calibration was made by connecting one of the reaction vessels, containing a measured quantity of 0.1N iodine solution, to the manifold system at point (1). Some vacuum is put on the system, and the reaction vessel stopcock is closed. The manifold-manometer system is then purged thoroughly with hydrogen sulfide at least four times. The system is then closed off with approximately 1 atmosphere of hydrogen sulfide in the manometer and manifold. The manometer is read accurately, and, without any delay, the stopcock is opened to the reaction vessel, then closed. The manometer is then read again, quickly. Without disconnecting the vessel it is shaken thoroughly to react all hydrogen sulfide with the iodine. If all the iodine has been used up, too much hydrogen sulfide has been taken into the tube, and the experiment must be repeated, using a smaller quantity of hydrogen sulfide, as shown on the manometer.

The excess iodine solution is titrated with thiosulfate (previously checked against the iodine) and the amount of hydrogen sulfide introduced into the reaction vessel is calculated. The measured volume of hydrogen sulfide will then give the volume of other gases it is desired to measure, such as oxygen, air, carbon dixoide, etc., for the same points on the manometer.

As to the precision of this method of gas addition, we might consider the following:

In the equation $P\ V = n\ R\ T$, $R$ and $T$ are constant. If there is no variation in $V$, the amount of gas measured ($n$) will be given by the pressure difference. There will, however, be some change in volume due to the change in level of the mercury in the manometer. If the free space in the manifold is relatively large compared to the volume change in the manometer free space, the fractional change in volume will be small, and satisfactory precision can be obtained at any position on the manometer. This precision is improved, however, by using readings at approximately the same position on the manometer. Whenever any change in this system is made, such as a new manometer, repair of manometer or manifold, change in the rubber tubing connection, or moving of the equipment, new calibrations must be carried out.

*Lubrication of the Reaction Vessel*

The reaction vessel has two points where careful lubrication is required: the large standard taper joint, and the stopcock. Since, in any study on corrosion, extremely small quantities of foreign substances might be effective as inhibitors, it is very important that the lubricant have little effect as an inhibitor, and the substance which appears most useful for this purpose is petrolatum.

In order to get the proper melting point characteristics, a mixture was made up using a micro-crystalline wax. 30 per cent micro-crystalline wax and 70 per cent refined petrolatum is quite satisfactory, and this is the mixture now being used, at 40° C.

*Charging the Reaction Vessel*

In charging the reaction vessel, the liquids are added first, using care not to wet the ground surface. The weighed metal strip is then placed in the vessel, and the vessel connected immediately to the manifold.

The vessel is rapidly evacuated by means of the electric pump, and filled with prepurified nitrogen. This procedure is repeated four times, shaking the tube after each evacuation.

If it is desired to admit a known quantity of oxygen (as air), as we are doing in further investigations, the gas pressure is not permitted to reach one atmosphere on the final filling with nitrogen, but is adjusted to a point based on the amount of air it will be desired to add. If air is added, the amount is then measured on the manometer, using the calibration as described above.

The tube is then filled with nitrogen, admitting the nitrogen to a pressure 50 mm. above atmospheric.

*Treatment of the Vessel after Charging*

The completely charged vessel is then secured tightly. The stopcock is secured by means of rubber bands or springs, and the large taper joint is secured by means of springs. The vessel is then placed in the constant temperature cabinet for the time desired, during all of which period the tube is rotated end to end at the desired speed. 15 rpm. is used in the work reported in this paper.

*Procedure at the Completion of a Run*

The vessel is emptied, and the metal strip is rinsed thoroughly under the tap. It is scrubbed with water and detergent, using a soft brush. It is rinsed thoroughly, scrubbed again, rinsed, dipped in alcohol, and

| System | Figure | Table | Mineral Spirits | Brine 4% NaCl | Inhibitor Kind | Inhibitor Ppm. by vol. of brine |
|---|---|---|---|---|---|---|
| I | 5 | I | 30 ml. | 30 ml. | None | None |
| II | 6 | II | 30 ml. | 30 ml. | A | 20 |
| III | 7 | III | 30 ml. | 30 ml. | A | 50 |
| IV | 8 | IV | 30 ml. | 30 ml. | A | 100 |
| V | 9 | V | 30 ml. | 30 ml. | B | 50 |
| VI | 10 | VI | 30 ml. | 30 ml. | B | 100 |
| VII | 11 | VII | None | 60 ml. | None | None |
| VIII | 12 | VIII | None | 60 ml. | A | 20 |
| IX | 13 | IX | None | 60 ml. | A | 50 |
| X | 14 | X | None | 60 ml. | B | 50 |

air-dried. Immediately after thorough drying the metal strip is weighed.

### REACTION VELOCITY EXPERIMENTS, IN ANEROBIC SYSTEMS

The tabulation on p. 380 shows the composition of the various reaction systems studied, all of which were free of oxygen. In each case the brine contained 1000 parts per million by volume of acetic acid, and there was one atmosphere of nitrogen present in the reaction vessel.

The various reaction systems were charged into the reaction vessels as described above. The vessels were then rotated in the constant temperature cabinet at 40° C. at 15 rpm., for various lengths of time, with a weighed test strip, as described above. At the end of the time for the particular experiment the tube was emptied, the test strip cleaned, and reweighed. For each experiment a new test strip was used, together with an entirely new charge for the tube.

The above table shows that the experimental data for a given system are tabulated in Tables I to X, inclusive, where the table numbers correspond to the system numbers.

The first column in each table represents the time ($t$) at which the metal strip was removed and weighed; the second column represents the

TABLE I.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 31.5 | 0 |
| 2 | 4.6 | 21.6 | 0.378 |
| 4 | 8.7 | 12.8 | 0.900 |
| 8 | 10.8 | 8.3 | 1.335 |
| 16 | 14.4 | 0.6 | 3.95 |

TABLE II.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 31.5 | 0 |
| 4 | 3.7 | 23.5 | 0.293 |
| 8 | 2.9 | 25.2 | 0.223 |
| 16 | 5.1 | 20.5 | 0.425 |
| 24 | 7.2 | 16.0 | 0.678 |
| 48 | 9.2 | 11.7 | 0.996 |
| 72 | 9.8 | 10.4 | 1.109 |
| 96 | 11.5 | 6.8 | 1.533 |

TABLE III.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 31.5 | 0 |
| 4 | 1.5 | 28.3 | 0.104 |
| 8 | 2.1 | 27.0 | 0.153 |
| 16 | 3.0 | 25.0 | 0.231 |
| 24 | 3.9 | 23.1 | 0.311 |
| 48 | 5.2 | 20.3 | 0.438 |
| 72 | 6.7 | 17.1 | 0.610 |
| 96 | 7.1 | 16.2 | 0.663 |

loss in weight ($w$) of the metal strip in milligrams at the indicated time. The third column gives the milligrams of solute (acetic acid) in the solution (the aqueous portion) at the time ($t$). The significance of the function ln ($x_0/x$) will appear below.

TABLE IV.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 31.5 | 0 |
| 4 | 1.0 | 29.3 | 0.068 |
| 8 | 1.2 | 28.9 | 0.086 |
| 16 | 2.0 | 27.2 | 0.148 |
| 24 | 3.2 | 24.6 | 0.255 |
| 48 | 3.5 | 24.0 | 0.270 |
| 72 | 6.1 | 18.4 | 0.536 |
| 96 | 6.3 | 18.0 | 0.560 |

TABLE V.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 31.5 | 0 |
| 4 | 2.2 | 26.8 | 0.157 |
| 8 | 6.1 | 18.4 | 0.536 |
| 16 | 8.1 | 14.1 | 0.802 |
| 24 | 11.4 | 7.0 | 1.502 |
| 48 | 13.4 | 2.7 | 2.45 |

TABLE VI.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 31.5 | 0 |
| 4 | 2.4 | 26.3 | 0.182 |
| 8 | 4.1 | 22.7 | 0.329 |
| 16 | 6.0 | 18.6 | 0.525 |
| 24 | 9.5 | 11.1 | 1.044 |
| 48 | 12.6 | 4.4 | 1.97 |

TABLE VII.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 63.0 | 0 |
| 4 | 3.4 | 55.7 | 0.122 |
| 8 | 6.5 | 49.0 | 0.287 |
| 16 | 10.1 | 41.3 | 0.419 |
| 24 | 13.2 | 34.6 | 0.599 |
| 48 | 20.8 | 18.3 | 1.233 |

TABLE VIII.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | ln ($x_0/x$) |
|---|---|---|---|
| 0 | 0 | 63.0 | 0 |
| 4 | 1.7 | 59.3 | 0.058 |
| 8 | 2.0 | 58.7 | 0.072 |
| 16 | 3.2 | 56.1 | 0.113 |
| 24 | 4.3 | 53.7 | 0.161 |
| 48 | 5.8 | 50.5 | 0.223 |
| 72 | 8.1 | 45.6 | 0.322 |

TABLE IX.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | $\ln (x_0/x)$ |
|---|---|---|---|
| 0 | 0 | 63.0 | 0 |
| 4 | 1.5 | 59.8 | 0.049 |
| 8 | 1.6 | 59.5 | 0.058 |
| 16 | 2.5 | 57.6 | 0.086 |
| 24 | 3.1 | 56.3 | 0.113 |
| 48 | 4.2 | 54.0 | 0.157 |
| 72 | 4.7 | 52.9 | 0.174 |

TABLE X.

| $t$ (hours) | $w$ (mg.) | $x$ (mg.) | $\ln (x_0/x)$ |
|---|---|---|---|
| 0 | 0 | 63.0 | 0 |
| 4 | 2.7 | 57.2 | 0.095 |
| 8 | 7.0 | 48.0 | 0.270 |
| 16 | 7.3 | 47.3 | 0.285 |
| 24 | 11.1 | 39.1 | 0.476 |
| 48 | 16.2 | 28.1 | 0.811 |
| 72 | 24.5 | 10.3 | 1.805 |

(*To be continued*)

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# IMPROVED PROCEDURE FOR INTERCONNECTING PIECE-WISE SOLUTIONS

BY

GABRIEL KRON [1]

SYNOPSIS

When a physical system, or its topological model, is torn apart into $n$ independent parts, a factorized form of the true inverse matrix $\mathbf{Z}$ may be established by finding the inverse of $n + 1$ much smaller matrices. No additional matrix multiplications are needed. The first $n$ matrices $\mathbf{Z}_1$ represent the $n$ subdivisions, while the $(n + 1)$-th matrix $\mathbf{Z}_4$ represents a newly created physical system, to be called "intersection" system. The latter matrix is constructed out of the fragments of the inverted subdivisions. This paper shows that it is not necessary to establish the $\mathbf{Z}_2$ and $\mathbf{Z}_3$ matrices that couple $\mathbf{Z}_4$ of the intersection system with $\mathbf{Z}_1$ of the $n$ subdivisions. The role of the coupling matrices can be taken over by the already established connection matrix $\mathbf{C}$. Thus the amount of storage and the clerical work are greatly decreased from that shown previously. Moreover the number of multiplications needed to evaluate the unknown $\mathbf{E}$ for a given $\mathbf{I}$ is reduced. A simple numerical example is also worked out by the improved procedure.

## TEARING *VERSUS* PARTITIONING

When a physical system can be described by a set of linear equations $\mathbf{I} = \mathbf{YE}$, the problem arises to express $\mathbf{E}$ as a function of $\mathbf{I}$, such as $\mathbf{E} = \mathbf{ZI}$, where $\mathbf{Z}$ may be either the true inverse of $\mathbf{Y}$, or some equivalent (or factorized) form of the true inverse. Such equivalent inverse forms may be established in many ways. Two of the most obvious procedures are as follows:

(1) The equations $\mathbf{I} = \mathbf{YE}$ of the entire system are written down and the resulting matrix $\mathbf{Y}$ is *partitioned*.

(2) The equations of the system are *not* written down. Instead, the physical system itself (or a topological model of it) is *torn apart*.

In tearing apart the physical system *new constraint forces appear* at the points of tear. (No such additional forces, nor other new physical concepts can appear during partitioning of a matrix.) These additional forces (analogues of the Lagrangean multipliers) simplify the necessary labor of computation and eliminate the large number of matrix multiplications needed in conventional partitioning.

A quicker and more systematic determination of these new forces liberated by tearing and of their field of action (the "intersection" network) is the main subject-matter of this paper.

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### COMBINATORIAL TOPOLOGY

The combined theory of original and additional forces in *discrete* physical structures requires the mathematical apparatus of "algebraic topology," or "combinatorial topology" (as opposed to the theory of *continuous* fields treated in "point-set topology"). For practical engineering purposes however, the theory and terminology of "electric circuits" can satisfactorily replace—at least in linear problems—the abstract concepts of algebraic topology, which is understood by very few specialists only (3, 4).[2]

In order to use the terminology of electric circuits, the "graph," or topological model of the physical system is reinterpreted as an electrical network in which the unknown **E** represent the junction potentials and the known **I** represent currents impressed at the junction-points (or vice versa). (The network does not have to be physically realizable.) However, it must be emphasized that the concept of impedance **Z** is not yet an integral part of Combinatorial Topology. It is the appearance of the impedance concept that requires the introduction of "tensors" into topology and it is the nature of the **Z** matrix that determines the manner of tearing and the manner of solving for the unknown quantities. That is, *the "network" of the engineer implies more concepts than the "graph" of the topologist.*

### THE $n+1$ COMPONENT SYSTEMS

The previous paper under consideration (1) solved the $\mathbf{I} = \mathbf{YE}$ equation of a physical system which can be represented by a network with a special structure. (After the network was torn into four parts, *each subdivision still was connected to the ground.* That is, the four subdivisions had one common point.) The paper developed detailed steps for the solution of such networks. The purpose of the present paper is to show that many of the steps included previously can be omitted in a purely numerical solution.

In particular, the purpose of this present paper is to show that in tearing apart the special type of network under consideration into $n$ subdivisions, it is sufficient to establish only $n + 1$ inverse matrices:

(1) the inverse $\mathbf{Z}_1$ of each of the $n$ subdivisions, and
(2) the $(n + 1)$-th inverse $\mathbf{Y}^4$ of the intersection network

The latter matrix contains as many rows and columns as there are cut tie-lines (that is, number of new constrained forces). It is constructed out of the fragments of the other $n$ inverted matrices. As a matter of fact, *this* $(n + 1)$*-th matrix need not be inverted.* The relevant equations can be solved by partitioning or other methods, without destroying the physics of the tearing process.

In the previous paper a long explanation was necessary to establish two additional matrices $\mathbf{Z}_2$ and $\mathbf{Z}_3$ that showed how the $n$ subdivisions

[2] The boldface numbers in parentheses refer to the references appended to this paper.

are coupled to the $(n + 1)$-th intersection system. In the present paper it will be shown that *these coupling matrices are superfluous*. Their role can be taken care of by the already established connection matrix **C**.

#### UNCHANGED STARTING STEPS

The following starting steps explained in (1) remain unchanged:

(1) The network is torn into $n$ independent subdivisions.
(2) The overhanging tie-lines (pigtails) are removed, since zero currents flow in them.
(3) Each of the subdivisions **Y** is inverted to form $\mathbf{Z}_\alpha$, $\mathbf{Z}_\beta$, etc.
(4) The inverted **Z**'s are arranged along the diagonal of a matrix $\mathbf{Z}_1$.

#### STEPS TO BE OMITTED

Since the coupling matrices $\mathbf{Z}_2$ and $\mathbf{Z}_3$ (between the subdivisions $\mathbf{Z}_1$ and the intersection network $\mathbf{Z}_4$) are not needed, the steps for the construction of the "augmented" inverse matrices $\mathbf{Z}_a'$ are all omitted. So is the section on "Scissors and Scotch Tape." In general, Eqs. 3 to 16 may be ignored. (However they still do have definite and useful physical meanings and they are going to be put to use in the *analytical* solution of the same problem.)

#### STEPS TO BE ALTERED

The omission of $\mathbf{Z}_2$ and $\mathbf{Z}_3$ means that the original subdivisions (the "primitive" system $\mathbf{Z}_1$) will not be interconnected to form **Z** of the original system. Instead, out of the original "primitive" system another, but much smaller "primitive" system will be formed, whose interconnection will bring to life the more restricted "intersection" network.

The labor-saving process thus consists of constructing next the **Z** of the "intersection" network, in place of the **Z** of the "original" network.

#### "INTERSECTION" NETWORK

The $\mathbf{Z}_4'$ of the $(n + 1)$-th network (Fig. 1*b*) whose construction is now to be undertaken, represents also the impedance matrix of the original network without any alterations whatever. The network however is viewed not from the large number of junction points (junction-pairs), but from the far fewer number of cut boundary points through which the constraint currents **i** circulate. The original network, when viewed from this internal, mesh point of view will be called "intersection" network (Fig. 1*b*). (From a geometrical point of view it represents the intersection space of the "augmented" original subdivisions.)

Four steps are made in constructing the $\mathbf{Z}_4'$ of the final intersection network out of the $\mathbf{Z}_1$ of the "primitive" original network (Fig. 2):

(1) The non-boundary rows and columns of $\mathbf{Z}_1$ are ignored and only the boundary axes are retained. The retained matrices will be

called "reduced" matrices. (In (1) the matrices of $\mathbf{Z}_1$ were enlarged, not reduced.) These reduced matrices represent the "primitive" intersection network $\mathbf{Z}_1'$ (a set of star-shaped networks in the present example, shown in Fig. 1*a*).

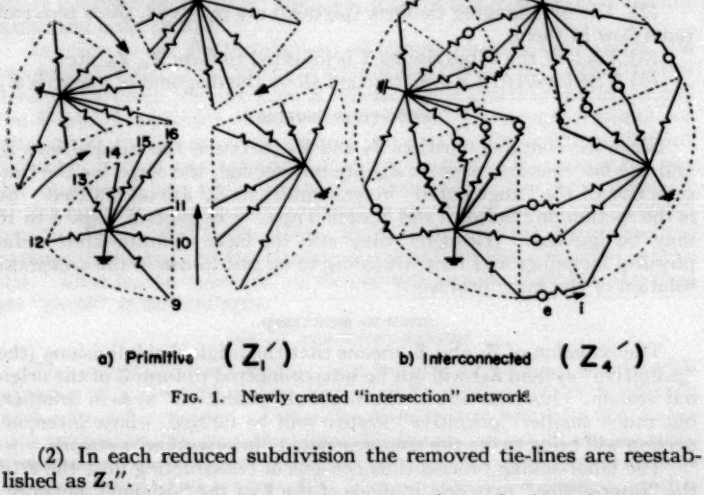


FIG. 1. Newly created "intersection" network.

(2) In each reduced subdivision the removed tie-lines are reestablished as $\mathbf{Z}_1''$.

(3) The tie-lines are interconnected to form the boundary system $\mathbf{Z}_4$.

Each of these three steps is going to be represented by a transformation matrix $\mathbf{C}_1$, $\mathbf{C}_2$ and $\mathbf{C}_3$.

(4) The last step consists of adding the impedances of the removed tie-lines to the diagonal elements of $\mathbf{Z}_4$ (Fig. 1*b*) to form $\mathbf{Z}_4'$.

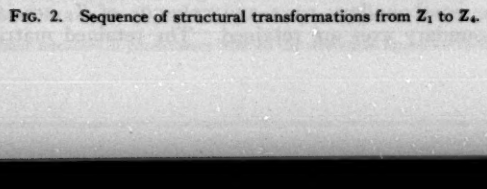


FIG. 2. Sequence of structural transformations from $\mathbf{Z}_1$ to $\mathbf{Z}_4$.

"PRIMITIVE" INTERSECTION NETWORKS

Considering the inverted matrix $\mathbf{Z}_\alpha$ of the first subdivision, its "reduced" portion is, by Eq. 4 of (1)

$$\mathbf{Z}_\alpha' = \begin{matrix} \\ 9\text{–}16 \end{matrix}\begin{matrix} 9\text{–}16 \\ \end{matrix}\begin{bmatrix} \\ \mathbf{Z}_1 \end{bmatrix}. \tag{1}$$

As the pig-tails are attached to axes 9 to 16, only the latter axes are retained and axes 1 to 8 are ignored.

The four reduced matrices are arranged along the diagonal of a matrix to form the "primitive" $\mathbf{Z}_1'$ of the intersection network.

The omission of the rows and columns of the first subdivision $\alpha$ may be represented by the transformation matrix

$$\mathbf{C}_{1\alpha} = \begin{matrix} \\ 1\text{–}8 \\ 9\text{–}16 \end{matrix}\begin{matrix} 9\text{–}16 \\ \end{matrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{2}$$

The reduction of all four subdivisions may be represented by the diagonal matrix $\mathbf{C}_1$

$$\mathbf{C}_1 = \begin{bmatrix} \mathbf{C}_{1\alpha} & & & \\ & \mathbf{C}_{1\beta} & & \\ & & \mathbf{C}_{1\gamma} & \\ & & & \mathbf{C}_{1\delta} \end{bmatrix} = \begin{matrix} \\ 1\text{–}8 \\ 9\text{–}16 \\ \vdots \end{matrix}\begin{matrix} 9\text{–}16\cdots \\ \end{matrix}\begin{bmatrix} 0 & & & \\ I & & & \\ & 0 & & \\ & I & & \\ & & 0 & \\ & & I & \\ & & & 0 \\ & & & I \end{bmatrix} \tag{3}$$

so that the "primitive" intersection network may be found from the "primitive" original network either by inspection, or by

$$\mathbf{Z}_1' = \mathbf{C}_{1t}\,\mathbf{Z}_1\,\mathbf{C}_1. \tag{4}$$

RE-ESTABLISHING THE TIE-LINES BY $\mathbf{C}_2$

If the removed tie-lines have a complex nature (that is, if two or more cut tie-lines are attached to one junction-point) they are re-established by the $\mathbf{C}_{2\alpha}$. One such $\mathbf{C}$ is established for each subdivision. In the example of (1), $\mathbf{C}_{2\alpha}$ is a unit matrix since each junction point had only one tie-line.

The four $\mathbf{C}_{2\alpha}$ to $\mathbf{C}_{2\delta}$ of the four subdivisions may be combined into one larger diagonal matrix $\mathbf{C}_2$. The resulting $\mathbf{Z}_1''$ matrix is now identical with $\mathbf{Z}_1'$.

INTERCONNECTING THE SUBDIVISIONS BY $\mathbf{C}_3$

After the removed tie-lines have been replaced by the aid of $\mathbf{C}_2$, the next step is to interconnect the tie-lines of the four subdivisions into

the resultant system. That was performed by $\mathbf{C}_b$ of Eq. 16 in (1), to be called here $\mathbf{C}_3$.

The two connection matrices $\mathbf{C}_2$ and $\mathbf{C}_3$ may be combined into one connection matrix

$$\mathbf{C}' = \mathbf{C}_2\,\mathbf{C}_3. \tag{5}$$

Of course it is possible to establish $\mathbf{C}'$ in one step, especially when $\mathbf{C}_2$ has a simple form.

### FINAL INTERSECTION NETWORK

The "reduced" matrices of the four subdivisions are interconnected by $\mathbf{C}'$ to produce $\mathbf{Z}_4$ of the intersection network

$$\mathbf{Z}_4 = \mathbf{C}_1'\,\mathbf{Z}_1'\,\mathbf{C}', \tag{6}$$

where $\mathbf{Z}_1'$ is the primitive intersection network. The $\mathbf{Z}_4$ is given in Eq. 20, in (1).

The remaining step consists of adding the diagonal matrix $\mathbf{Z}_8$ of the removed tie-lines (Eq. 21, (1)) to $\mathbf{Z}_4$. The result is $\mathbf{Z}_4'$. (This step can be performed by $\mathbf{C}_4$ but it is simpler to add $\mathbf{Z}_8$ outright.)

The resultant intersection network is shown in Fig. 1*b*. It is a *mesh* network, while the original physical system is a *junction* network.

The inverse of $\mathbf{Z}_4$ is calculated as $\mathbf{Y}^4$. (This inversion however, need not be performed.)

### "COMPLETE" CONNECTION MATRIX

The intersection network without the tie-lines, namely $\mathbf{Z}_4$, could have been established directly from $\mathbf{Z}_1$ (instead of $\mathbf{Z}_1'$) by defining the *complete* connection matrix $\mathbf{C}$

$$\mathbf{C} = \mathbf{C}_1\,\mathbf{C}_2\,\mathbf{C}_3 \tag{7}$$

so that

$$\mathbf{Z}_4 = \mathbf{C}_t\,\mathbf{Z}_1\,\mathbf{C}. \tag{8}$$

Similarly the coupling matrices $\mathbf{Z}_2$ and $\mathbf{Z}_3$ may be established from $\mathbf{Z}_1$ (see Eq. 18, (1)) by

$$\begin{aligned} \mathbf{Z}_2 &= \mathbf{Z}_1\,\mathbf{C} \\ \mathbf{Z}_3 &= \mathbf{C}_t\,\mathbf{Z}_1. \end{aligned} \tag{9}$$

Hence *by constructing the complete* $\mathbf{C}$ *matrix the construction of* $\mathbf{Z}_2$ *and* $\mathbf{Z}_3$ *may be dispensed with.*

### CALCULATION OF E

For any given $\mathbf{I}$ the voltages $\mathbf{E}$ are found by Eq. 26, in (1)

$$\mathbf{E} = (\mathbf{Z}_1 - \mathbf{Z}_2\,\mathbf{Y}^4\,\mathbf{Z}_3)\mathbf{I}. \tag{10}$$

Substituting $\mathbf{Z}_2$ and $\mathbf{Z}_3$ in terms of $\mathbf{Z}_1$ and $\mathbf{C}$

$$\mathbf{E} = (1 - \mathbf{Z}_1 \mathbf{C} \mathbf{Y}^4 \mathbf{C}_t)\, \mathbf{Z}_1 \mathbf{I}. \tag{11}$$

Hence $\mathbf{E}$ may be found by the following series of six steps involving the product of a matrix by a vector (2)

$$\begin{array}{l|l} \mathbf{E}_1 = \mathbf{Z}_1 \mathbf{I} & \mathbf{i}^1 = \mathbf{C}\,\mathbf{i} \\ \mathbf{e} = \mathbf{C}_t \mathbf{E}_1 & \mathbf{E}_2 = \mathbf{Z}_1 \mathbf{i}^1 \\ \mathbf{i} = \mathbf{Y}^4 \mathbf{e} & \mathbf{E} = \mathbf{E}_1 - \mathbf{E}_2. \end{array} \tag{12}$$

It should be noted from Fig. 1*b*, that the voltages impressed in series with the torn boundary coils are $\mathbf{e}$, while the resultant currents in the boundary coils are $\mathbf{i}$. The equation of solution of the intersection mesh network is thus

$$\mathbf{i} = \mathbf{Y}^4 \mathbf{e}. \tag{13}$$

The corresponding equation of state $\mathbf{e} = \mathbf{Z}_4 \mathbf{i}$ may be solved by any convenient method, without having the true inverse $\mathbf{Y}^4$ available.

For all purposes the inverse "table" of the original equation $\mathbf{I} = \mathbf{YE}$ may be represented by the diagonal matrix with five elements

$$\mathbf{Z} = \left[\begin{array}{cccc|c} \mathbf{Z}_{1\alpha} & & & & \\ & \mathbf{Z}_{1\beta} & & & \\ & & \mathbf{Z}_{1\gamma} & & \\ & & & \mathbf{Z}_{1\delta} & \\ \hline & & & & \mathbf{Y}^4 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{Z}_1 & \\ \hline & \mathbf{Y}^4 \end{array}\right]. \tag{14}$$

The complete connection matrix $\mathbf{C}$ is also part of $\mathbf{Z}$.

The storage of these five small matrices and of $\mathbf{C}$ serves the same purpose for almost all contingencies, as the storage of the true inverse. In particular the five inverse matrices may be used for:

(1) calculating $\mathbf{E}$ for any arbitrary $\mathbf{I}$,
(2) calculating "factorized" inverses for still larger systems constructed by interconnecting the present system with other systems, and
(3) making small changes in any part of the system.

### NUMERICAL EXAMPLE

Let the numerical example in (1) be repeated with the improved procedure.

The system of Fig. 14 of (1) with equations $\mathbf{I} = \mathbf{YE}$ is torn apart into two parts. The two identical subdivisions and the removed pigtails are shown in Fig. 15. The $\mathbf{Y}_1$ and $\mathbf{Z}_1$ matrices of each subdivision are given in Eqs. 32 and 33.

The $\mathbf{Z}_4$ matrix of the intersection network is established in four steps:

(1) The "reduced" matrix includes only axes **3** and **4** of $\mathbf{Z}_a$. Two of these matrices are arranged along the diagonal of a matrix.

$$\mathbf{Z}_1' = \begin{array}{c} \\ 5' \\ 6' \\ 5'' \\ 6'' \end{array} \begin{array}{c} \begin{array}{cc} 5' & 6' \end{array} \quad \begin{array}{cc} 5'' & 6'' \end{array} \\ \left[\begin{array}{cc|cc} 7 & 3 & & \\ 3 & 7 & & \\ \hline & & 7 & 3 \\ & & 3 & 7 \end{array}\right] \end{array} \frac{1}{15}. \tag{15}$$

(2) Since each boundary junction-point contains only *one* pigtail, $\mathbf{C}_{2a}$ is the unit matrix and so is $\mathbf{C}_2$.

(3) The interconnected system is shown in Fig. 17 with the new variables $i^5$ and $i^6$. The connection matrix is by Eq. 39, in (1).

$$\mathbf{C}_3 = \begin{array}{c} \\ 5' \\ 6' \\ 5'' \\ 6'' \end{array} \begin{array}{c} \begin{array}{cc} 5 & 6 \end{array} \\ \left[\begin{array}{cc} -1 & \\ & -1 \\ \hline 1 & \\ & 1 \end{array}\right] \end{array} = \begin{array}{c} \\ m' \\ m'' \end{array} \begin{array}{c} m \\ \left[\begin{array}{c} -1 \\ 1 \end{array}\right] \end{array}. \tag{16}$$

Multiplying the diagonal "reduced" matrices $\mathbf{Z}_1'$ by $\mathbf{C}_3$ (since $\mathbf{C}_2 = \mathbf{1}$), the $\mathbf{Z}_4$ of the intersection network is found by $\mathbf{C}_{3t}\,\mathbf{Z}_1'\mathbf{C}_3$ as

$$\mathbf{Z}_4 = \begin{array}{c} \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cc} 5 & 6 \end{array} \\ \left[\begin{array}{cc} 14 & 6 \\ 6 & 14 \end{array}\right] \end{array} \frac{1}{15}. \tag{17}$$

(4) Adding the impedances of the removed boundary coils, the final $\mathbf{Z}_4'$ matrix of the intersection network is given in Eq. 43 and its inverse $\mathbf{Y}^4$ in Eq. 44.

The factorized inverse table consists of two identical $\mathbf{Z}_{1a}$ and $\mathbf{Y}^4$ arranged along the diagonal.

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## NOTES FROM THE NATIONAL BUREAU OF STANDARDS

### PRECISION ETCHING OF GLASS SCALES

Information on a number of improved techniques for producing precise etched glass scales has recently been made available by the National Bureau of Standards.[1] Methods for the domestic manufacture of high-precision graduated glass circles and reticules for theodolites were developed by Raymond Davis and Chester I. Pope of the Bureau staff at the request of the U. S. Army Engineering Board. The techniques may also be applied to the manufacture of glass scales for optical measuring instruments such as dividing heads, indexing tables, cam-rise gages, and the like.

Theodolites equipped with glass scales were not manufactured in this country at the time the Bureau initiated its studies. Although the etching of glass has been widely practiced for a number of years, there was no information generally available for etching the high-precision glass circles required by theodolites. These circles are read by means of a micrometer microscope so constructed that a four-inch circle can be read to one second of angle. A graduation displaced by 0.00001 in. (0.25$\mu$) would cause an error of one second in the reading. Thus the precision required is extremely high and difficult to achieve.

In the course of its investigations, the Bureau developed two methods for producing very precisely graduated and numbered glass circles. The steps required by the basic method include coating the glass with a wax resist, engraving, etching, removing the resist, and filling the etched areas. The second technique is a rapid "photographic" procedure designed to produce replicate scales from glass circles produced by the basic method. In the photographic method, the glass circle, coated with a light-sensitive resist, is contact-printed with an optically flat master. The circle is then "developed" by applying a solvent which dissolves out the unexposed portions without attacking the exposed areas. The latter have been rendered insoluble by the action of light. The remaining operations are slightly modified from those of the basic method to compensate for differences in the light-sensitive resist.

#### RESISTS

A considerable portion of the Bureau's study was devoted to the formulation of a number of wax and light-sensitive resists. Although

[1] For further technical details, see "Techniques for Ruling and Etching Precise Scales in Glass and Their Reproduction by Photoetching with a New Light-Sensitive Resist," by R. Davis and C. I. Pope, NBS Circular 565, available from the Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 30 cents.

waxes are resistant to hydrofluoric acid, they cannot always be cleanly engraved. The addition of other materials such as rosin, resin, pitch, and asphalt improves the resist's engraving characteristics but may reduce adhesion to glass or resistance to acid. However, several resists were developed which are composed of both natural and synthetic materials and have none of the undesirable properties.

A resist can be applied in either of two ways. In the simpler method, the glass piece is set spinning on a whirler. Then the resist is heated and flowed onto the whirling glass piece. A more uniform coating, free from impurities or lint, is produced by the "pull-out" method, where the circle is submerged in a benzene-resist solution and withdrawn at a constant rate by mechanical means. Other factors affect the quality of a resist: A temperature change of 10 to 20° F. is likely to reduce the bond to the glass, and a resist will not adhere to a glass surface which is not carefully cleaned and dried. Comparisons showed that waterproofing the glass with amino silane not only prevents the resist from lifting during etching but also produces sharper lines.

### ENGRAVING

Two Bureau-designed engraving machines aided the development of the glass etching techniques. One is a reticule-ruling and measuring machine for engraving the glass circles. It was also used to test ruling properties of experimental resist formulations. The other instrument is a pantograph which numbers the graduations. Ruling and scribing tools used with these instruments are made from diamond or steel. The steel tools for engraving graduations are high-carbon steel drill rod hardened by heat-treating.

### ETCHING

After the coating on the glass is engraved, the glass piece is etched by gaseous hydrogen fluoride (either dry or moist) or with an aqueous solution of hydrofluoric acid. Gas etching produces a matte finish and is used principally for etching fine lines that will not be filled with pigment. Etching by gas requires a relatively longer time and is thus more easily controlled. Etching solutions produce a clear etch or a frosted etch, depending on their compositions. Those solutions giving a clear etch generally contain 48 to 60 per cent hydrofluoric acid; those which give a frosted etch contain salts which promote the formation of insoluble compounds on the glass surface. Concentrated hydrofluoric acid solutions are too fast to control when etching fine lines. However hydrofluoric and phosphoric acid combined produce a mild etching solution which can be controlled for etching very fine graduations.

### PHOTOETCHING

The "photographic" method developed by the Bureau prepares replicate glass circles from a master circle made by the usual procedure.

The photographic method is much faster since no mechanical engraving is required. In this method, a light-sensitive coating is applied to a glass circle which is then contact-printed with the master "negative" and "developed." The coating consists of phenol-formaldehyde resin with iodoform as a sensitizer. The addition of resorcinol-formaldehyde resin improves the light-sensitive qualities of the coating.

The light-sensitive resin is applied to the glass by the "pull-out" method with special precautions to ensure a very uniform, thin coating. A cabinet of special design was constructed for contact-printing a design on the resin-coated circle from the glass master. After exposure, the unexposed areas are dissolved out by a solution containing methyl ethyl ketone, butyl alcohol, and ethyl alcohol. The circle is then baked to increase the resistance of the developed resin coating to hydrofluoric acid. The best etching results are obtained with an aqueous mixture of hydrofluoric and phosphoric acids.

The Bureau also studied the effect of relative humidity on the deterioration of phenol-formaldehyde and cold-top enamel resists. Excellent results have been obtained with the resins developed for the photographic process. Graduation lines on the replicate circles are about 1$\mu$ wider than those on the master circle.

## LINEAR THERMAL EXPANSION OF TEFLON

As part of its basic research program on the properties of materials, the National Bureau of Standards has determined the linear thermal expansion of annealed polytetrafluoroethylene (Teflon) from $-$ 190° to $+$ 300° C.[1] This work, conducted by R. K. Kirby of the Bureau staff, also included a study of the effect of internal residual stresses on the expansion of Teflon and confirmed the presence of two first-order transitions at 20° and 30° C. With the data thus obtained, it is possible to calculate the dimensions that a Teflon product must have at room temperature in order that it will have the desired dimensions at the temperature of application.

Teflon is a thermoplastic resin made by polymerizing tetrafluoroethylene. It is being used in an increasing number of applications by industry because of its outstanding chemical and thermal stability, excellent electrical insulating properties, and low coefficient of friction. It has been used as gaskets, washers, and tubing by the chemical processing industry; high voltage and ultra-high frequency insulators in electronic equipment; and as oil-free bearings in mechanical equipment.

Because the long Teflon molecules are not straight over their entire length, the structure of Teflon is not completely crystalline. Above

[1] For further technical details, see "Thermal Expansion of Polytetrafluoroethylene (Teflon) from $-190°$ to $+300°$ C.," by R. K. Kirby, J. Research NBS 57 (1956) RP2696.

330° C. Teflon is free of strain and has no crystalline structure. On cooling, Teflon starts to crystallize at about 325° C., and becomes opaque and waxy in appearance. On further cooling first-order transitions occur at 30° and 20° C.

The Bureau studied the thermal expansion of four samples of annealed Teflon. The fused-quartz tube and dial indicator method was used on three of the samples. In this method the sample is placed upright in a fused-quartz tube, closed at the bottom. A movable fused-quartz rod rests on top of the sample and transmits the differential change in length of the sample and fused-quartz to a dial indicator fastened to the top of the tube. The expansion of a fourth sample was measured with precision micrometric apparatus, which indicated that the small force exerted on the other samples by the quartz rod and dial indicator did not affect their expansion. Dry ice and liquid air provided the low temperatures, and an electric furnace and heating cabinet the high temperatures.

The coefficients of linear thermal expansion of annealed Teflon at 0° and 50° C. were found to be $114 \times 10^{-6}$ and $118 \times 10^{-6}$/° C., respectively. The average coefficients of linear expansion from 0° to 25° C. and from 25° to 50° C. were $200 \times 10^{-6}$ and $124 \times 10^{-6}$/° C., respectively. The large differences between the "instantaneous" and average coefficients of expansion are caused by the transitions at 20° and 30° C. Repeated measurements indicate that the error of these coefficients is about $3 \times 10^{-6}$/° C.

The investigation also demonstrated that residual stresses affect the length and thermal expansion of Teflon. When Teflon is stretched or elongated, internal tensile stresses are set up which decrease its expansion. When compressive stresses are present in Teflon, it is shorter and its expansion is increased. Annealing Teflon by heating to 350° C. and cooling slowly removes the internal residual stresses.

### AN AUTOMATIC MICRO-IMAGE FILE

A micro-image data storage and retrieval device recently developed at the National Bureau of Standards provides rapid access to any one of 10,000 information-containing frames recorded in miniature on a 10-in.-square sheet of microfilm. The instrument operates on a continuous basis; it automatically searches the microfilm and photographically prints out one frame every 2 seconds. Designed and built by M. L. Kuder of the Bureau's electronic instrumentation laboratory, the device is intended for use in other Government agencies.

The machine is particularly applicable where large volumes of data must be assembled in a predetermined sequence from a master random file. Information may be in the form of pictures, drawings, fingerprints, sets of numbers, letters, or other symbols, or even single stages

of electronic circuit diagrams. Quantity and kind of data are limited only by the size of the individual frame ($\frac{1}{10}$-in. square) and the photographic resolution of the film emulsion. Although the basic storage capacity of the machine is for a 10,000 frame matrix, the matrix can be interchanged with others from a static file.

Input to the machine is from a perforated teletype tape containing the coded locations of the desired frames in the order in which they are to beprinted out. The assembled data produced by the machine come out on a 10-in. wide strip of photo-sensitive paper of any required length. Individual frames are enlarged to $\frac{1}{2}$-in. squares. Commercial automatic developing equipment processes the photographic paper.

The instrument is essentially a combination of digital computer electronic circuitry and a pair of precision servomechanisms that search $X$ and $Y$ axes of the matrix. The location of the desired frame is fed into a 20-bit (binary digit) register from the teletype tape. The register consists of a capacitor memory and coincidence identification circuitry. The first 10 bits recorded in the register control the $Y$ position selection while the second 10 bits control the $X$ position.

The matrix is supported on a drum 10 in. in diameter and is fastened at one edge with dowel pins to insure its accurate location on the drum. The drum is servocontrolled in both linear and rotary axes of motion, corresponding to the $X$ and $Y$ axes of the matrix. The servos that shift the matrix to the chosen coordinates are mechanically coupled with precision gearing to two code commutators.

The code commutators, one associated with each axis, control the coordinate positions to which the matrix is located. These commutators are photo-etched with 100 ten-bit numbers corresponding to the standard teletype binary bit code. The two particular positions on the commutators are selected by a serial mechanical search with contacting brushes until a code combination is found that matches the binary bits recorded in the 20-bit register. Magnetic clutches and brakes provide rapid starting and stopping of the drum with uniform over-travel in locating every position on the matrix. A single induction motor supplies all motive power to the machine.

At the beginning of the cycle of operation, a teletype tape reader reads a 4-decimal-digit number into the 20-bit register in terms of a binary-digit code. A space symbol is customarily inserted in the teletype tape following each 4-digit number. On detecting this space symbol, the machine's program control stops the tape reader, engages the magnetic clutches on the $X$ and $Y$ servos, and looks for the compatible code on the two coordinate axes. When the compatible code is found, the clutches disengage and magnetic brakes stop the drum. A print lamp is briefly turned on to make a photographic exposure of the selected microfilm frame on the photosensitive paper. When the exposure is completed, the teletype tape advances to the next instruc-

tion, the drum returns to its zero position, and the machine proceeds with the next search cycle.

Fifteen successive frames are printed in a row across the 10-in. width of the print paper by means of a step positioning mirror. This mirror performs a function similar to the character spacing on a typewriter: it automatically advances the image one space on the photographic paper for each printout. Upon completion of a line, a line-feed servo advances the paper a fixed amount.

The instrument recognizes two other symbols, the "carriage return" and the "line feed." These symbols instruct the machine to return the step positioning mirror to its zero position, and to advance the paper one line. Whenever these functions are desired, they can be inserted into the teletype tape.

Although the machine was primarily designed as an outscriber for obtaining programmed printing from a large file of negatives, it can temporarily be set up as an inscriber to prepare its own matrices of 10,000 frames each. Using the same machine to prepare a matrix insures that each frame will be accurately located whenever it is subsequently used.

# THE FRANKLIN INSTITUTE

## Minutes of the Stated Meeting

### October 17, 1956

Following the Annual Medal Day Reception and Dinner held in Franklin Hall, the October Stated Meeting of the Institute was called to order by Mr. S. Wyman Rolph, President, at 8:15 p.m., in the Lecture Hall. 280 members and guests were in attendance.

The President announced that the minutes of the May Stated Meeting had been printed in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE; there being no corrections or additions, they were approved as published.

Dr. Burlew, at the request of the President, spoke a few words on "Coming Events at the Institute."

Following Dr. Burlew's remarks, the President told the members that the world-wide activities in honor of Benjamin Franklin during this 250th Anniversary year had been even more widespread than first hoped, and that even those at the Institute were stunned by the great outpouring of affection and respect from practically all the peoples of the world. The Institute was proud that it was the clearing house and fountainhead of this great birthday party, still being carried on.

Then followed the formal presentation of medals, after which the President introduced Dr. Edwin H. Land who gave a most interesting talk on studies in one-step photography, with the reproduction of many interesting photographs.

The meeting adjourned at 9:57 p.m.

H. V. BAIL
*Secretary*

## JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in this JOURNAL within the next few months:

CHANG, S. S. L.: Transient Effects of Supply and Connecting Conduits in Hydraulic Control Systems

STEINMAN, D. B.: The Design of the Mackinac Bridge for Aerodynamic Stability

ABRAHAMS, HAROLD J. AND MARION B. SAVIN: Benjamin Rush's Course in Chemistry at the Young Ladies' Academy

MARIN, JOSEPH: Mechanical Properties of Materials for Combined Stresses Based upon True Stress and Strain

HODGE, P. G., JR.: Piecewise Linear Isotropic Plasticity Applied to a Circular Cylindrical Shell with Symmetrical Radial Loading

PIPES, LOUIS A.: Analysis of Electric Circuits Containing Nonlinear Resistance

HILL, E. L.: Electromagnetic Radiation from Lightning Strokes

SLEPIAN, JOSEPH: The Magneto-Ionic Expander Isotope Separator

## LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 287 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, Fridays and Saturdays; and 2 P.M. until 10 P.M. on Wednesdays.

### RECENT ADDITIONS

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## MEMBERSHIP

### ACTIVE MEMBERS ELECTED BY THE BOARD OF MANAGERS

**October 17, 1956**

ACTIVE FAMILY

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Robert B. Jennings
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Laurence T. McCurdy '47
Mrs. Randal Morgan '36
C. W. Oakford '36
Guy C. Pierce '36
Milton C. Radler '55
George T. Sharp '56

## PUBLICATIONS RECEIVED

The Editors wish to call attention to these new books which have been received in the JOURNAL office for review. Lack of space prevents publication of full reviews.

WEATHER ANALYSIS AND FORECASTING. VOLUME II. WEATHER AND WEATHER SYSTEMS, by Sverre Petterssen. Second edition, 266 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $6.00.

COMMUNICATION ENGINEERING, by W. L. Everitt and G. E. Anner. Third edition, 644 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $9.00.

EXPERIMENTAL PHYSICAL CHEMISTRY, by Farrington Daniels, Joseph Howard Mathews, John Warren Williams, Paul Bender and Robert A. Alberty. Fifth edition, 482 pages, diagrams, 6 × 9 in. Price, $6.50.

INTEGRAL TRANSFORMS IN MATHEMATICAL PHYSICS, by C. J. Tranter. Methuen's Monographs on Physical Subjects. 133 pages, 4 × 6½ in. London, Methuen & Co. Ltd.; New York, John Wiley & Sons, Inc.; 1956. Price, $2.00.

ORDINARY NON-LINEAR DIFFERENTIAL EQUATIONS, by N. W. McLachlan. Second edition, 271 pages, diagrams, 6 × 9¼ in. New York, Oxford University Press, 1956. Price, $5.60.

DICTIONARY OF PHOTOGRAPHY, edited by A. I. M. Sowerby. Eighteenth edition, 719 pages, diagrams, 4¼ × 7¼ in. New York, Philosophical Library; London, Iliffe & Sons, Ltd.; 1956. Price, $10.00.

THE GENERATION OF ELECTRICITY BY WIND POWER, by E. W. Golding. 318 pages, illustrations, 5½ × 8½ in. New York, Philosophical Library, 1956. Price, $12.00.

APPLIED ELECTRICAL MEASUREMENTS, by Isaac F. Kinnard. 600 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1956. Price, $15.00.

DETERMINATION OF ORGANIC COMPOUNDS, by H. G. Stone. 233 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $5.00.

PHYSIQUE MATHÉMATIQUE CLASSIQUE, by Théodore Vogel. 216 pages, 4½ × 6½ in. Paris, Librairie Armand Colin, 1956. Paper, no price.

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NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES
FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

# MAGNETIC PROPERTIES OF SOME IRON AND IRON OXIDE MICROPOWDERS*

BY

R. B. CAMPBELL, H. AMAR, A. E. BERKOWITZ AND P. J. FLANDERS

The theory of highly dilute ferromagnetic micropowders treats these powders in first approximation as a set of Stoner-Wohlfarth assemblies (1).[1] In each of these assemblies, the particles are, by definition, randomly oriented single-domain non-interacting ellipsoids of revolution with the same axial ratio, $m$, volume, $v$, and magnetization density, $I_0$.

The agreement with experiment is only qualitative; for example, the predicted value of the coercivity is usually too high, although of the correct order of magnitude. The quantitative discrepancy is, however, substantially decreased by a more rigorous computation of the theoretical values. The following scheme is suggested:

(*a*) Determine, through careful measurements on the electron micrographs, $n(m,v)dmdv$, the fractional number of particles with elongation and volume in the respective ranges $(m,m + dm)$ and $(v,v + dv)$. The volume fraction of particles with elongation, $m$, is then

$$g(m) = \mathbf{S}_v n(m,v) \cdot v, \tag{1}$$

where **S** stands for integration or summation.

(*b*) Starting from the Stoner-Wohlfarth (1) and Rhodes (2) tables, derive the "magnetization matrix" $F(H,m)$, yielding the magnetization curve for an $m$-assembly. The magnetization curve of the actual powder is then given by

$$\frac{I_H}{I_0} = f(H) = \mathbf{S}_m F(H,m) g(m). \tag{2}$$

In Eq. 1, one should delete from the $m$-$v$ plane of integration the area

* This note describes work performed under the cooperatively sponsored Magnetics Research Program at The Franklin Institute Laboratories for Research and Development. A technical paper on this topic was presented at the Conference on Magnetism and Magnetic Materials, October 16, 17, and 18, 1956 at Hotel Statler, Boston, Mass., sponsored by the American Institute of Electrical Engineers in cooperation with the American Physical Society, The American Institute of Mining and Metallurgical Engineers, and the Institute of Radio Engineers.

[1] The boldface numbers in parentheses refer to the references appended to this note.

corresponding to multi-domain and superparamagnetic particles as suggested by Bean (3).

It will be shown in general that the predicted value of the coercivity is lowest when:

(*a*) the whole distribution (and not simply its median) is considered, and
(*b*) the contribution of the superparamagnetic and multi-domain particles is taken into account.

Measurements have been made on a series of dilute iron and iron oxide micropowders in an effort to correlate their magnetic and structural properties. The magnetic measurements include magnetization and hysteresis curves for various temperatures, coercive force and remanence as functions of applied field, and preliminary torque data. The structural measurements include particle counts from electron micrographs to determine size and shape distributions, crystallite size, and composition. It has been found that when the method outlined above is rigorously applied, very satisfactory agreement is obtained between the measured and predicted values of magnetic parameters. Such deviations as do occur between the experimental and predicted values can probably be attributed to two principal factors. One is the interaction between particles; the other involves the hysteresis of multi-domain particles containing only a few domains. The contributions of these factors are being examined.

Coercive force and remanence have been measured in the range from room temperature to liquid nitrogen temperature on samples of magnetite micropowders. These data have been obtained with and without an applied field during cooling. A change of slope in the coercive force *versus* temperature curve occurs at the cubic orthorhombic transformation temperature ($\sim -$ 160° C.). The increase in coercive force in the orthorhombic magnetite is in accord with the crystal anisotropy measurements of Williams, Bozorth and Goertz (4), although crystal anisotropy does not appear to determine the coercive force in the cubic region. The remanence curve obtained with a field applied during cooling shows a similar increase in slope at the transformation temperature. This behavior apparently corresponds to the establishment of an easy axis near the field direction in accordance with the mechanism proposed by several investigators.[2]

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[2] Reference 4 contains references to the previous work.

## NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, M.D., FOUNDER WILLIAM G. BATT, D.Sc., DIRECTOR

**Seminar on Instrumentation for the Transplantation of Nucleoli.**[1]—Dr. M. J. Kopac of the Department of Biology, Graduate School of Arts and Science, New York University, addressed the Seminar of the Biochemical Research Foundation on Wednesday afternoon, June thirteenth.

*The nucleolar lesion:* The increasing prominence of nucleoli (enlargement, multiplicity, and pleomorphism) in human ovarian neoplasms is a striking indication of a relationship, though as yet unqualified, between nucleolar variability and degree of malignancy (1). One of the features frequently shown by nucleoli in both human and amphibian tumor cells (2) is the tremendous pleomorphism. A characteristic pattern in the large, pleomorphic nucleoli is the appearance of vacuoles. These may vary in size, number, and position. All vacuoles may be of the same size in some nucleoli, while in others there is considerable variation in size. The large pleomorphic nucleolus with extensive vacuolization, either internal or marginal, is a good example of the nucleolar lesion.

Other types of nucleolar lesions include such things as: fragmentation, multiple nucleolation, conversion to highly dendritic or filamentous structures, production of Feulgen-positive substances, development of eosinophilic inclusions, extrusion of eosinophilic substance into nucleus and later into cytoplasm (2), excessive nucleolar-associated DNA, production of pyroninophilic inclusions with a vacuolar configuration, extrusion of entire nucleolus into cytoplasm, extrusion of nucleolar vacuoles or nucleolonema into cytoplasm, etc.

An important problem is to determine the relationship between the nucleolar lesion and the cancer process. Although much has already been learned about nucleolar lesions from studies on morphologic and cytochemical changes, there is more to be learned by direct experimentation. Through the application of refined micrurgical procedures, nucleoli can be transposed (from nucleus to cytoplasm of same cell) or transplanted (from nucleus to cytoplasm of different cell). In this way, the effects of environment on the nucleolus can be demonstrated. Similarly, the effects of the nucleolus on the new environment can also be established. Thus, we have the basis for evaluating the "subcellular ecology" of intracellular structures. Numerous technical difficulties

[1] These investigations are supported in part by a research grant (C-2018) from the National Cancer Institute of the National Institutes of Health, Public Health Service, and by grant DRG-271 from the Damon Runyon Memorial Fund for Cancer Research, Inc.

had to be solved before these operations became feasible; also several improvements in micrurgical instrumentation had to be made.

*Transplantation procedures:* One of the most important factors for increasing the success of subcellular transplantations is to eliminate water from the environment of cells during micrurgy. For this, the fluorocarbon-transfer procedure was developed (3, 4). The frequent coagulation of cytoplasm and its adherence to the glass walls of a micropipette can be avoided, in most instances, by first coating the pipettes with a thin silicone film (5). By preventing coagulation of the cytoplasm or other subcellular fractions, one also prevents drastic changes in the sample before it is injected into another cell. In critical experiments, measured volumes of subcellular material are drawn into a siliconized micropipette, and the sample is sealed within the micropipette at both ends by fluorocarbon seals consisting of small volumes of Kel-F-I oil (polymer of trifluor-monochlorethane). These seals protect the sample from chance contamination with aqueous media and possible injury. The outer seal, injected with the sample, serves as a marker and thereby indicates that the material was actually injected into the cell.

*Instrumentation:* An important advance in micrurgy is the volumetric submicromanipulator (6, 7) for removing or implanting known volumes of subcellular substances into living cells.

Another advance is the television micromanipulator, using the RCA Vidicon closed circuit television. This instrument, equipped with a monochromator and quartz or reflecting optics, may be used with either visible or ultraviolet light (8). The present instrument contains four new Leitz micromanipulators which may be loaded with as many as eight microneedles or micropipettes, although ordinarily only four are used, one on each micromanipulator. Two semiquantitative volumetric controls are used for extracting nucleoli from cells with a micropipette and then transferring these structures to other parts of a cell or to different cells.

The television micromanipulator has now been augmented with a Conrac 17-in. auxiliary viewer and a fast rise cathode ray oscilloscope. The oscilloscope, Tektronix type 545 with type 53/54C dual trace preamplifier, has a flexible delay circuit. Any one of the 525 horizontal lines comprising the video image can be selected and displayed on the oscilloscope. At the same time, a pulse from the positive gate is fed into the viewer to brighten and thereby indicate the line selected.

*Design of experiments:* The notation of symbolic logic and calculus of sets can be profitably applied towards the analysis of nucleoli and other subcellular structures (8). This enables one to set up the important factors and their more probable relationships. As an example: $n_1 \epsilon N_1 \underline{\subset} C_1$ to $n_1 \epsilon C_1 - N_1$, states that the nucleolus, $n$, is an element, $\epsilon$, of the nucleus, $N$, which is a subgroup, $\underline{\subset}$, of the cytoplasm, $C$, and that

it is transferred so that it now becomes an element of the cytoplasm but not, $-$, of the nucleus. The subscript, 1, identifies the cell and since it is the same in both, the operation is transposition, that is, the nucleolus is transferred from the nucleus to the cytoplasm of the same cell.

On the other hand, $n_1 \epsilon N_1 \underline{\subset} C_1$ to $n_1 \epsilon C_2 - (n_2 \epsilon N_2)$, indicates transplantation. The nucleolus from the nucleus of cell 1 is transferred to the cytoplasm of cell 2. The statement shows that the nucleolus, following transfer, is an element of the cytoplasm of cell 2, but not of the nucleus which still has its own nucleolus, $n_2$. Furthermore, the statement: $n_1 \epsilon N_1 \underline{\subset} C_1$ to $Qn_1 \epsilon C_2 - N_2$, states that part of the nucleolus, $Qn_1$, from cell 1 is transplanted to the cytoplasm of cell 2 from which the nucleolus, $n_2$, was previously removed.

Another example is: $n_2 v n_1 \underline{\subset} C_1 - N_1$. This statement shows that the nucleolus from cell 2 (transplantation) and, $v$, nucleolus from cell 1 (transposition) are a subgroup of the cytoplasm of cell 1 but not of the nucleus. This is a way of comparing the effects of transplantation and transposition on nucleoli or on the cytoplasm. In this example, both nucleoli are subjected to the same cytoplasmic environment, although the cytoplasm may be subjected to the action of two similar, but also possibly different, nucleoli.

Transposition experiments, therefore, can serve as controls for transplantation experiments. This is the most direct means of controlling or evaluating the possible effects of genetic constitution on either the nucleolus or the action of the nucleolus on its new environment. Many variations of transposition and transplantation experiments, involving the nucleolus, are possible and each one may be precisely stated, using symbolic notation. There is always recorded the source and position of the nucleolus prior to the operation as well as its position following the operation.

*Demonstration of nucleolar lesions:* Many of the manifestations of nucleolar lesions are morphologic of which some may be clearly seen in living cells with phase contrast illumination or in cells stained with hematoxylin-eosin or methyl green-pyronin Y. Certain specialized structures, especially the nucleolonema require silver impregnation techniques (9). Hypertrophy, fragmentation, pleomorphism, or vacuole formation can be observed in living cells with ordinary light.

In living cells, the more satisfactory scheme for demonstrating the development of morphologic nucleolar lesions, following subcellular transplantation, is to employ sequence photography (intervals ranging from 5 to 30 minutes between photographs of same cell or field). Such a series of pictures will show whether morphologic changes have occurred following experimental treatment of the cell. Obviously, the cells must be maintained under optimal conditions favorable to cellular survival and growth.

Other manifestations of nucleolar lesions are cytochemical or functional or both. One effect that may be studied quantitatively is the rather specific supravital staining of nucleoli with methylene violet (Bernthsen). Changes in retention of dye or stainability can be readily detected and measured. For this, the television micromanipulator interconnected with a delayed sweep cathode ray oscilloscope is indispensable.

For example, a nucleolus previously stained supravitally with methylene violet is transposed or transplanted with the television micromanipulator (direct optical or video monitoring). Immediately after the operation is completed, the preparation is illuminated with light of contrasting color (ca. 580 mμ). A scan line is then selected that passes through the center of the nucleolus (under optical magnification of 200 ×, a nucleolus in the frog kidney cell covers 15 to 45 scan lines). From the amplitude and width of the displayed pulse on the oscilloscope and changes that may occur in such pulses, one may clearly recognize anything unusual in dye retention following the micrurgical transfer of a nucleolus to a new environment.

*Further developments in instrumentation:* Other instrumentation and techniques need to be developed to permit greater approaches to perfection. For example, a new four-place micromanipulator, now being built by Jack Harris, will be controlled by remote synchro-drive mechanisms. An automation control unit is included whereby certain micrurgical operations such as positioning can be programmed and hence automatically performed. Included in this instrument is an entirely new inverted microscope which embodies, among other things, the flying spot principle (for ultraviolet light) plus a remotely controlled fine adjustment and mechanical stage. Scan line selection and electronic computing devices will be used with this instrument for analyzing changes in nucleolar structures and their physico-chemical properties following transposition and transplantation. The new micromanipulator-microscope can also be controlled manually, except that motion will be induced and controlled by push buttons—rather than by levers, control knobs, or other schemes used heretofore.

Although present methods and equipment are adequate for removing nucleoli from one cell and inserting them into another, there is, however, room for improvement. The removal of a structure like a nucleolus is a special problem. There is required the rapid application of suction over a limited, adjustable volume. To accomplish this, we are experimenting with magnetostrictive volume controls. It is expected that this type of micropipette, if successful, will be integrated with the new micromanipulator now being constructed.

SUMMARY

These new research instruments and procedures will help clarify answers to such questions as the origin and significance of nucleolar

lesions. It is yet to be established whether these nucleolar lesions are the cause or the result of malignancy. Transplantation experiments, coupled with transposition experiments as controls, show promise of yielding informative answers to these fundamental questions.

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MICRORECORDING, by C. M. Lewis and W. H. Offenhauser, Jr. 456 pages, photographs and drawings, 6 × 9 in. New York, Interscience Publishers, 1956. Price, $8.50.

Apropos of the current hustle to mechanize commercial operations from beginning to end, these authors have undertaken the subject of information storage and retrieval. The scale of the problem which they treat is comparable to the task which a large business organization encounters in establishing a record retention program. Those faced with the establishment of a system for recording and storing the items of business which must be filed would surely be helped by Lewis' and Offenhauser's experience.

This experience has been very broad, more so than that which the engineer would normally encounter. For example, information is given covering the legality of these records under particular circumstances. A list is given by states and countries showing what proof is needed when original documents have been lost. Laws governing the copying of certain items such as currency and special documents are also given.

Naturally it is hard to describe the popular systems and system components such as cameras, readers and other instruments without referring to them by trade names. Hence, there is considerable reference to trade-named devices as well as the principles they employ. The trade names thus become a part of technical jargon and have a shorthand value in the discussion.

Besides those sections devoted to planning costs and administration the authors have devoted much effort to detailing the technical mechanics of available equipment. In this the reviewer tests their up-to-dateness by looking for, and finding described, the new Eastman "Minicard" system. This equipment is actually still in the laboratory, but will undoubtedly some day be the most advanced microrecording method. The Eastman system is unique in its ability to store the document record and its classification code together. The reduction in storage volume is one thousand to one; that is, the contents of one thousand file cabinets can be stored in a single cabinet of Minicards.

Eastman and others have contributed to the preparation of this book by permitting the authors to reproduce whole sections of previously published copy in the form of plates. In a very large appendix the American Standards Association has permitted numerous papers pertaining to lenses, enlargers and other related components to be reproduced also as plates. The authors have tied all of this information together reasonably well and have provided very large bibliographies with international reference coverage.

It may be concluded that the book is valuable in relating what exists. On the other hand, developers of future systems may find little help here in projecting analytically their thoughts for improvement of contemporary methods. Imagination and analysis of the basic limitations of present methods are needed to promote forward strides in the field of information storage and coding. These authors have not introduced such an analytical attack. But these remarks are not intended as adverse criticism, for there is no pretense at presenting an engineering treatise; and a very practical book has resulted. This work evidences extensive first-hand experience on the part of the authors and the advice given is therefore authoritative.

C. W. HARGENS

APPLIED GROUP-THEORETIC AND MATRIX METHODS, by Bryan Higman. 454 pages, 6 × 9¼ in. New York, Oxford University Press, 1955. Price, $9.60.

This book, as its name implies, is concerned with the applications of Group Theory. The fields of application are primarily physics and chemistry, and since the book is designed for students in these fields, there is little rigorous mathematics, definition, lemma, theorem variety to be found in it.

There have been books of this type before. For example: H. Weyl's "The Theory of Groups and Quantum Mechanics," and even Speiser's "Theorie des Gruppe von endlicher Ordnung" (both of which are now classics) touch on topics to be found discussed in the

book under review. However, no previous book has the broad scope of this one. This will be evident when we describe the contents in greater detail.

Although, as we have mentioned, the reader will not find very much rigorous mathematics here, the student of pure mathematics will nevertheless find reading this book a profitable experience. He will find material here that will give him deeper insight into what lies behind (and what has occasionally provided the impetus for the development of) the theorems and lemmas of pure mathematics. For example, consider Chapter XII which deals with continuous groups. It begins by showing how continuous groups are a generalization of finite groups, and then proceeds to develop, in a well motivated discussion, representations and characters in continuous groups, infinitesimal operators and the group manifold, differential operators and Hilbert space, etc.

The first half of the book deals with the theory of finite groups and its application to a description of the external forms of crystals, the internal structure of crystals, the vibration of molecules, and to the problem of factor analysis. The last half of the book is concerned with the theory of continuous groups and its application to relativity theory, quantum theory, molecular structure and spectra, and Eddington's quantum relativity.

The reader will find in this book a wealth of material presented in an extremely lucid style.

H. L. PLATZER

RANDOM PROCESSES IN AUTOMATIC CONTROL, by J. Halcombe Laning, Jr. and Richard H. Battin. 434 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $10.00.

During the formative years of the development of basic techniques for the design of servomechanical systems, emphasis was placed on steady-state situations with respect to signal inputs and their responses. But today, after fifteen years of intensive development of automatic control systems, understanding the random processes associated with the input signals to the automatic control systems is just as important as the detailed design of the hardware for the system.

Since 1951 the authors have given a course at MIT in random processes in automatic control, principally on fire control and navigation systems. The reader, therefore, is privileged to read for the first time much material which has never been generally available, due to military security regulations and the fact that the material is new with the authors.

The foundations of any study pertaining to random processes must rest on probability theory and on the techniques for applying such theory to the design of complex instruments. The subject book treats both aspects of the problem, first presenting material which forms the theoretical background, pertaining to the basic concepts of probability and random time functions; then it applies this material to the analysis and design of linear control systems whose parameters are either constant or time-varying.

Eight chapters and ten appendices form the body of the book. The first four chapters, lucidly written, include detailed material on the basic concepts of engineering probability. The authors' introductory remarks emphasize the importance of statistical concepts where random variations in signals give rise to interference effects and where an automatic control system is called upon to perform many tasks, each with a wide range of variables, instead of a single simple one. Point functions, simple statistical properties and interrelations, distribution functions, correlation and power spectrum relations, and stationary time phenomena, with many linear and non-linear applications, are treated in this part of the book.

The second part of the work treats: the analysis of a class of time-invariant linear systems having as inputs a stationary random process; the practical evaluation of the rms error criterion of performance for time-varying linear systems in response to time stationary on non-stationary inputs; optimum linear least squares smoothing and prediction for stationary random processes (methods of Wiener, Bode and Shannon, Wiener-Hopf, etc.); and operations with systems having input signals of finite length.

Since much of this material has not been available to a widely-read public, it is predicted that the techniques described will find ever-widening application to the design of non-military as well as military automatic control systems, directly as a result of the publication of this book. It is recommended

as required reading for every systems-minded engineer, as well as every design engineer associated with control systems. Needless to say, it should be an excellent textbook, there being a large number of problems in each chapter suitable for classroom assignment and home study. S. CHARP

REVIEW OF TEXTILE PROGRESS, Vol. 6 (1954). 586 pages, 5¼ × 8 in. Manchester (England), The Textile Institute and the Society of Dyers and Colourists, 1955. Price, 37/6 ($5.50).

This annual technical handbook has earned its deserved praise in international textile circles. The regular appearance of these reviews from year to year is significant to those associated with the science and technology of today's textiles.

In an effort to continue the challenging nature of this publication from year to year, several radical changes have been instituted. For the first time in Volume 6 there are contributors from outside Great Britain. The section on "Colouring Matter" has been written by a world authority, Professor Venkataraman of the University of Bombay. Another overseas contributor, Mr. S. B. McFarlane, of the Celanese Corporation of America has written the section on "Synthetic Fibers."

This chapter is noteworthy for it is characterized by a wide coverage of the different aspects of this ever-growing subject. Then too, it is right that the author of this section should be an American, for the United States is the world center of synthetic fiber production.

This particular volume of the review presents 13 new authors and 3 new subjects. For both the textile technologist and the textile scientist, this present edition is vital as a handy reference to the new developments not only in the fiber field but also in the textile processing industries from the conversion of fibers into yarns through weaving, knitting, lace and embroidery. The section on "Narrow Fabrics" is an innovation. Unfortunately it reviews the progress of the past decade rather than concentrating on present day manufacture. Nevertheless, this presentation is worth reading especially since in few textile references are narrow fabrics discussed.

Due to the breadth of subject matter plus the current data presented by each authority this annual review serves as a record of progress. Of unusual value to not only the scientist and the technologist but also the writer, the lecturer and the graduate student, are the references noted at the bottom of each page. These footnotes are in themselves exceedingly useful as a guide for further study. CAROLINE BOYER

FOUNDATIONS: DESIGN AND PRACTICE, by Elwyn E. Seelye. v.p., diagrams, 8 × 10 in. New York, John Wiley & Sons, Inc., 1956. Price, $16.00.

"Age should speak, and multitude of years should teach wisdom." Consideration should be given to this engineering book written by a man with over 40 years practice in his subject. His three-volume authoritative book for civil engineers has had two editions; he is an active member of numerous top engineering groups; and he is senior partner in a consulting engineering firm.

*Foundations* was written for those engaged in construction work including buildings, bridges, dams, roads, airports, pipe lines, tunnels, in fact any type of sub-surface construction, under water or land. This book is well illustrated with diagrams, maps, photographs, and includes a wealth of up-to-date tables, charts, and data on those subjects treated. "Discussions" and "Red Lights" in each of its 19 sections, together with well-illustrated "Right-Wrong," "Difficulties—Proposed Remedy" sections, make this workable indexed volume a real contribution to the library of reference books for the architect, design engineer and those engaged in responsible construction positions.

EARL R. BOWMAN

SCIENCE AND INFORMATION THEORY, by Leon Brillouin. 320 pages, diagrams, 6 × 9 in. New York, Academic Press, Inc., 1956. Price, $6.80.

Few books have been printed on this subject. The National Science Foundation advanced a grant to the author to complete his book, which is based on a series of lectures originally delivered at the International Business Machines Corporation and later at different universities.

The theory about which he writes is a mathematical one, based upon probability considerations and precise definitions. It

concerns only the number of possible answers to a problem under different circumstances independent of the observer, not the "value" of this information relative to a particular observer. The fact that under some circumstances there may be a multiplicity of answers to a problem gives rise to the need for some unique way of representation. The logarithm of the number of possible answers is taken to be the quantity of information. This is shown to be the equivalent of the quantity "entropy" so familiar to classical physicists as a measure of the lack of information or of disorder. Under certain circumstances it becomes desirable to make use of the negative of entropy, or negentropy.

The twenty chapters of the book can be grouped into six major divisions: basic definitions and applications; coding principles and problems and error detection and correction; analysis of sampled data; the entropy and negentropy of thermodynamical systems and random motions in physical systems; the negentropy principle of information applied to heat engines and observations in various physical systems; and the negentropy principle applied to telecommunications, channel capacity, reading and writing, computers, and other methods of organizing physical data.

Dr. Brillouin has completed a magnificent task in presenting his subject matter with a simplicity and clarity of expression which will be absorbed readily by members of the engineering profession. Several chapters will be of especial interest to the designers of out-of-the-ordinary measuring instruments and of course those who are concerned with various computing and information processing systems. Dr. Brillouin's contribution to the field of information theory is an important one, for it will help bridge the gap between a theory, which in many cases has not been well understood because it may not have been well presented, and its engineering and scientific applications. While not specifically so intended, the book could well serve for a one-semester graduate course in the subject.

S. CHARP

SOLID STATE PHYSICS, VOLUME 2, edited by Frederick Seitz and David Turnbull. 468 pages, diagrams, 6 × 9 in. New York, Academic Press, Inc., 1956. Price, $10.00.

The second volume in this series achieves a better balance between experimental and theoretical topics than did the first. The presentation of the material in the present volume is uniformly excellent. It is quite evident that this series will be indispensable to any student or investigator interested in the physics of the solid state.

The first three chapters deal with some of the more recently developed techniques for obtaining structural information on the atomic scale about solids. These chapters are: "Nuclear Magnetic Resonance," G. E. Poke; "Electron Paramagnetism and Nuclear Magnetic Resonance in Metals," W. D. Knight; and "Applications of Neutron Diffraction to Solid State Problems," G. G. Shull and E. O. Wollan. The two articles concerning nuclear magnetic resonance present concise treatments of the theoretical background and experimental techniques in this field, as well as the results and interpretation of representative investigations. The article on neutron diffraction first summarizes those aspects of neutron scattering which are pertinent to solid state problems. Then follows a detailed presentation of the results of the application of this technique to a wide variety of crystal and molecular structures. Particular emphasis is placed on studies of magnetic materials.

The fourth article is "The Theory of Specific Heats and Lattice Vibrations" by J. de Launay. This chapter is a comprehensive presentation of the various models and computational methods used in the analysis of lattice dynamics. Comparisons with experiment are discussed and a special section on the diamond lattice is included.

The final chapter, "Displacement of Atoms during Irradiation" by F. Seitz and J. S. Koehler is a quantitative assessment of the status of this increasingly important field. The procedure is to emphasize those aspects of radiation damage which can be produced under the most carefully controlled conditions. The phenomena are then discussed using the fundamental concepts of atomic and solid state theory. For this reason, the material is of interest to any solid state investigator since it is approached as a study of imperfections in solids. The chapter deals principally with the effects of light charged particles since the experiments with such particles can generally be more carefully controlled.

AMI E. BERKOWITZ

## BOOK NOTES

THE AMERICAN ARBACIA AND OTHER SEA URCHINS, by Ethel Browne Harvey. 298 pages, plates, 6¼ × 9¼ in. Princeton, Princeton University Press, 1956. Price, $6.00.

The American sea urchin, plentiful along the East Coast of North America, is the subject of this well-organized treatise. The animals are used in experiments on cells, particularly at the Marine Biological Laboratory in Massachusetts.

Since the *arbacia* experiments have been in progress since 1892, there is a wealth of data available. The author has systematically arranged her material in four parts. Part I gives the history of sea urchins (beginning prior to Aristotle) and their use as food and medicine, with a general survey of their natural history (feeding, growth, breeding, etc.), and their classification. Part II is devoted to the normal development of the sea urchin and treats all phases of the egg structure and growth. Part III deals with centrifuged eggs, describing methods and procedures in great detail. The final part is a compilation of experimental work, arranged alphabetically by subjects (62) from "Ageing of Eggs" to "Yolk Granules."

For anyone working in the fields of experimental embryology and cell physiology, this carefully compiled work should prove invaluable.

STRUCTURAL GEOLOGY, by L. U. de Sitter. 552 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $8.00.

The author of this advanced text is a distinguished Dutch geologist, Professor of Structural and Applied Geology at the University of Leiden.

The book can only be appreciated by a trained geologist. It is primarily concerned with systematizing the existing knowledge in order that genetic relationships may be distinguished. The aim of the book is to establish a link between the theory and practice of orogenesis.

Part I covers the physical properties of rocks (Theoretical Structural Geology); Part II discusses comparative structural geology; and Part III discusses theories concerning the origin of some of the largest structural units (geotectonics).

The book contains much material unfamiliar to most geologists, and it should prove a valuable addition to the geologist's library.

COALMINING, by I. C. F. Statham. 564 pages, illustrations, 5¾ × 8¼ in. New York, Philosophical Library, 1956. Price, $15.00.

Written by a Professor of Mining at the University of Sheffield, this volume presents a history of coal mining in Great Britain. Opening with a general chapter on the importance of coal mining in the British economy, the book then proceeds to cover all phases of the industry, from the origin and occurrence of coal and the planning of a colliery, through the administration of such a plant, to the actual methods used in mining (including prospecting, drilling the coal, transporting it underground, mine lighting and drainage). A chapter on safety and health problems and a chapter on "Welfare and Training in the Coal-mining Industry" conclude the book. The volume is well illustrated and it contains an index, but no bibliography.

ENGINEERING AS A CAREER, by Ralph J. Smith. 365 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1956. Price, $4.75.

Designed for use in a freshman orientation course, this book has six major objectives: (1) to acquaint beginning students with engineering as a career; (2) to indicate the qualifications, duties and responsibilities of an engineer; (3) to define the engineering profession in terms of functions as well as branches; (4) to provide motivation for the study of pre-engineering courses; (5) to provide training in the philosophy and technique of problem solution; and (6) to preview the basic engineering sciences and demonstrate their application.

This could well be read by high school students interested in engineering, before entering college, as well as by freshmen already enrolled. The text is designed for a two-unit course of about 32 class meetings, although it can be worked into either a one- or three-unit course.

THE CONDENSED CHEMICAL DICTIONARY, edited by Arthur and Elizabeth Rose. Fifth edition, 1200 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1956. Price, $12.50.

This new, much larger fifth edition of a standard chemical dictionary replaces the previous 1950 edition. The new version contains over 30,000 up-to-date entries, in 1220 double-column pages. The trade-mark and trade-name coverage has been greatly expanded, with new material supplied direct from the producers. In addition, the dictionary has been thumb-indexed and printed in larger type, both of which add to the convenience of its use.

The dictionary is a practical one for use by chemists and engineers, as well as purchasing agents and executives.

DESIGN OF PIPING SYSTEMS, by the Engineering Departments of The M. W. Kellogg Company. 365 pages, illustrations, 8¼ × 11 in. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1956. Price, $15.00.

Privately published in 1941 by the M. W. Kellogg Company, the first edition of this book represented the first adequately organized treatment of the analytical method for evaluating the components of irregular piping systems. The second edition, published jointly by John Wiley and Chapman & Hall, makes this method available to a considerably wider audience. At the same time, it augments the first edition by embodying the many changes in piping systems introduced since 1941. The structural phase of piping design is thoroughly covered in nine chapters and three appendices. New chapters on expansion joints and pipe supports have been added, and structural and fluid vibration is treated in a new, final chapter.

DYNAMIC FACTORS IN INDUSTRIAL PRODUCTIVITY, by Seymour Melman. 238 pages, 5¼ × 8¼ in. New York, John Wiley & Sons, Inc., 1956. Price, $4.75.

The author, aided by a grant from Columbia University, spent four years studying two phases of modern business, namely, the mechanization of production and the rise of administrative overhead. The book sets forth the factors which brought about changes in production methods and manpower utilization, and attempts to explain how these affect productivity levels.

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**First Atomic Submarine Engine Runs 66 Days at Full Power.**—Successful conclusion of an unprecedented power demonstration by an atomic ship propulsion reactor has been announced by the U. S. Atomic Energy Commission.

The Nation's first practical reactor power plant—the land-based prototype of the submarine *Nautilus* at the Commission's National Reactor Testing Station in Idaho—was routinely shut down on August 8, 1956, at the end of what is believed to have been the longest full power run ever completed by any type of propulsion plant—land, sea or air.

On a single charge of uranium fuel (and using only part of that charge), the nuclear reactor located at the desert test facility operated at an average power of 100 per cent for more than 66 days and 66 nights. The test was designed to prove the reliability and stamina of pressurized water reactors for ship propulsion. The prototype of the reactor now in the *Nautilus* met the most demanding requirements that could be placed upon it.

At the end of the 1600-hr. test, the reactor still had many hundreds of hours of full operation available and it has resumed operation at varying power levels to obtain engineering test data and to train the Navy's atomic sailors. These trainees, officers and men, will man future atomic power ships of the fleet.

The test demonstrated the feasibility of nuclear power ships cruising at high speeds and without refueling for periods of time hitherto unattainable. Among the important benefits of the test was the obtaining of essential and heretofore unavailable information on reactor performance during extended high power runs. These data will be used in the design of improved nuclear propulsion plants both for naval vessels and civilian nuclear power plants.

To indicate the significance and magnitude of the operation, it was pointed out that if the *Nautilus* itself had made a cruise for this length of time—1600 hours—she could have steamed at top speed, submerged, around the world and many thousands of miles more. For example, she could have left the New London, Connecticut, submarine base, steamed around Cape Horn, across the Pacific, across the Indian Ocean, around the southern tip of Africa, returning to the starting point; and then continued for an extensive Arctic cruise before the scheduled shutdown.

If such a cruise had been made by a diesel-power submarine of similar power, about 1,600,000 gallons of fuel would have been required. This amount of oil will fill 160 railroad tank cars, making a freight train more than one mile long.

During the 66 days and nights of the test, Navy men assigned to the Naval Reactors Facility at the Idaho installation manned the submarine prototype plant in round-the-clock watches just as if they were in the *Nautilus* on a cruise. While the Navy ran the nuclear plant, engineers of the Westinghouse Electric Corporation which developed and built the reactor, studied its performance and gathered data.

The history-making prototype first produced power in May of 1953. Since that time, it has been used to supply data and advance the technology of pressurized water reactors. It was refueled in March, 1956, after two and a half years operation on the original uranium charge.

**Dislocations and Plastic Flow.**—Dislocation theory has been developed over the past several decades to describe satisfactorily the mechanical behavior of metals. For example, this theory explains why metals are not nearly as strong as previous theoretical studies indicated they should be. However, evidence of the physical existence of dislocations has been, until recently, complex and indirect. This lack of direct evidence, coupled with the extreme versatility of the theory, led metallurgists to suspect that perhaps dislocations had no reality.

Several significant experiments have recently been reported which leave little doubt that dislocations are real and behave essentially as described by the theory. F. L. Vogel, W. G. Pfann, H. E. Corey and E. E. Thomas at Bell Telephone Laboratories observed that certain internal boundaries in germanium crystals, when properly etched, appeared as a series of discrete pits in the surface. They proved by microscopic and X-ray measurements that these pits marked the positions of dislocations.

Investigations of plastic flow in germanium crystals by F. L. Vogel and Miss L. C. Lovell at Bell Laboratories confirm the value of dislocations in explaining the behavior of crystalline solids. By deforming the crystals at high temperatures and cooling to "freeze in" the structural changes, they have been able to observe and study the movements of individual dislocations during both deformation and annealing.

For an ideally plastic material, the theoretically calculated dislocation density is inversely proportional to the radius of curvature of the neutral plane in a bent bar. Where this radius is large relative to the bar thickness, the density should be essentially uniform over the cross-section. Dislocations conformed to these expectations in bent-and-annealed bars, but did not in bent bars that were not annealed. In the latter, not only was the average dislocation density appreciably larger, but the number of dislocations increased from the neutral plane toward both the top and bottom of the bar. This is ascribed to residual elastic strain and to trapping of excess dislocations, both of which are removed by annealing.

The study of dislocation etch pits has grown rapidly in the last two or three years, and now work of this type is being undertaken in many different laboratories. At least ten materials have been successfully studied by this technique, and the number is continually expanding. All of this work is providing basic information on the behavior and structure of metal crystals which some day is expected to be translated into materials having improved and more accurately predictable characteristics.

**Engraved Combination Plates.**—A new method of etching half-tones and line copy on the same magnesium plate, announced by The Dow Chemical Company, brings substantial savings to quality magnesium photoengraving.

The combination-etch process, developed by Dow after extensive research in cooperation with the American Newspaper Publishers Association Research Institute, Inc., is

a new aspect of the popular Dow-Etch process.

For newspapers, the development means timelier news-photo coverage for any one edition; readers, in effect, will be brought even closer to the news as it happens.

The new technique is effected by first etching the magnesium combination plates to a depth of about 0.016 in. in a new combination bath. Then the plates are deep-etched in a conventional Dow Deep-Etch bath. This brings open areas of the combination plate down to 0.025 to 0.035 in.

A patent application has been filed on the new development.

According to Dow, proper etch technique in a balanced bath will produce a plate with:

1. Greater consistency of etch quality than ever before possible.
2. Virtually no undercutting.
3. Good depth in half-tone and highlight dropout areas.
4. Ample depth in small non-printing line areas.
5. At least 0.005 inch depth in 65 screen highlight areas.
6. Reasonably smooth etch in open areas.
7. Good highlight dot formation.

The combination etch technique can be used for ordinary half-tone or line etching as well as combination etching.

The combination technique is intended primarily for coarse 65–85 screen newspaper type combinations, Dow magnesium research engineers said. They added the process may not yet be entirely suited to some of the fine-screen combination copy now being engraved by commercial shops. However, Dow is continuing research efforts in this direction.

The Dow Etch Process, perfected some years ago, has been used successfully on both line and half-tone magnesium engravings. But its use in etching combination plates has been limited until now.

**Electrical Appliances in Automobiles.**—The Cornell-Dubilier Electric Corporation is introducing a new line of electrical power supplies known as "Powercons" designed specifically for the automotive market to convert the normal 6 or 12 volt d-c. battery power supply to 110 volt a-c. 60 cycle power similar to that found in the home.

Availability of a-c. power in a car will enable motorists to enjoy many important electrical appliances while on the road. Families who enjoy travel, camping and other outdoor activities can now take tape recorders, small TV sets, record players, shavers and other small electrical appliances with them to add new pleasure to their trips. Sales representatives, doctors and businessmen who use their cars regularly in business will find a "Powercon" installation invaluable if they wish to use dictating equipment. Electronics servicemen will find a tremendous advantage in having "commercial power" available at their service truck so that laboratory instruments may be used anywhere for trouble diagnosis or scientific investigation.

These small, compact, box-like units with various power ratings can be installed in convenient spaces such as under the dashboard or in a car's trunk compartment. They may then be wired to a convenient outlet such as at the cigarette lighter socket for quick, simple accessibility. Absolutely no alteration of the car's ignition system is required.

The Cornell-Dubilier line of "Powercons" for mobile service range in size, weight, and capacity from light 20 watt units for operating electric shavers to 150 watt or 200 watt units for operation of larger devices. All of the above "Powercon" units deliver 110-volts a-c. 60 cycle power. Several models have output voltage control built in.

**Ductile Iron in Reel Spiders.**—Ductile iron reel spiders are being used in the Superior Separator Company's new multiple-reel width graders, eliminating the breakage problem that once occurred with gray cast iron spiders, the Hopkins, Minnesota, firm reports.

Designed for screen cleaning barley, oats, durum and rye, the grading machine comes in four, six and eight-reel models. It grades grain at about 40 bushels per hour per reel. The ductile iron reel spiders are about 18 in. in diameter and 1 in. thick. Spaced about 18 in. apart on a steel shaft, they are connected by wrought steel lift arms. Since the reel spiders support the lift arms and the stainless steel grading screen, as well as the entire grain load during the grading operations, a tough, shock-resistant metal is required for the job.

Gray iron spiders had a record of field failures. Deflection in the length of the span between bearings caused the gray iron spiders to crack. Tough ductile iron, with its superior tensile and yield strength and high modulus of elasticity, proved to be the answer to this problem.

Soon after ductile iron replaced cast iron in reel spiders, the new metal went through an unplanned test that graphically demonstrated its toughness and shock resistance, the Superior Separator Company reports. Ductile iron spiders were being installed in a width grader located high up in a grain elevator. One of the spiders accidentally dropped from a height of 160 ft. while being hoisted up the side of the structure. It crashed down on a railroad track, bouncing several feet into the air before rolling to rest unharmed. When it was finally installed, the spider operated perfectly despite its 160-ft. plunge, according to Superior Separator.

**Mass Production of Transistors.**—A new technique for the mass production of high-speed and ultra-high-frequency transistors, announced by Philco Corporation, represents a major advance in semiconductor technology. Two new types of transistors, a Micro-Alloy Transistor (MAT) and a Surface-Barrier Diffused Transistor (SBDT), the direct result of this technique, are now in engineering development.

Developed at Philco's Research Laboratories in cooperation with the Navy, Army Signal Corps and the Air Force, the new devices will be produced by the Lansdale Tube Co., a division of Philco, at its new semiconductor plant at Spring City, Pa. The MAT and SBDT may be used in electronic "brains," guided missiles, communications, radar, and other military equipments.

The MAT and SBDT devices are fabricated from the element germanium and utilize Philco's unique Surface Barrier technique. This process, now highly mechanized, permits the mass production of transistors requiring precision control of critical dimensions. It avoids the necessity of manually assembling the germanium and the electrode material and alloying them in a furnace.

Primarily designed for high-speed electronic computers, the MAT employs a new Philco "microalloying

process" which uses doped semiconductor layers only a few millionths of an inch thick. In conventional alloying, the doped layer must be made relatively thick, and due to the basic nature of the alloying process, is difficult to control. The new device is reported to be at least ten times faster than the fastest vacuum tube in electronic computers.

The other new device, the SBDT, will operate in the ultra-high-frequency range of 500 megacycles. It will find its widest use in the 20 to 200 megacycle range. This transistor, which also utilizes Philco's electrochemical and microalloying techniques, permits the effective use of what is known as "surface diffusion." In this particular process, the tiny germanium transistor blanks are subjected to a very carefully controlled "atmosphere" at high temperatures. This atmosphere contains particles of metal in gaseous form. When properly controlled, atoms of the metal penetrate the surface of the germanium by diffusion. The SBDT technique, using highly mechanized precision etchings, results in transistors having Alpha cutoff frequencies above 600 megacycles.

The new MAT and SBDT semiconductors will join a group of Philco transistors that runs the gamut from milliwatt hearing aid amplifiers to 10 watt power units, in addition to 40 to 80 megacycle Surface Barrier Transistors used in computers, communications and military equipments.

**Trailmarkers for Arctic.**—Electronic techniques for marking safe trails in the arctic have been developed by the Army's Engineer Research and Development Laboratories, Fort Belvoir, Virginia.

A system consisting of two parallel wires and a vehicular-mounted radio-type receiver has been successfully tested on the Greenland Ice Cap. An alternating current is fed into the wires, which are buried beneath the snow on either side of the trail, to electrically mark the route. The receiver on a tracked vehicle commonly known as the "Weasel" detects the current in the wires. Indicators in the vehicle give the driver his position within the trail. Warning devices alarm the driver when the vehicle gets out of bounds and crosses a trail wire.

Poor visibility during the Polar night, snow storms and summer "white outs," (dense arctic fog) make free movement over the ice cap virtually impossible. Travelers face the possibility of getting lost and falling into hidden crevasses. Bridged over slightly with snow, crevasses are dangerous even in good visibility. With weak wind-built snow bridges concealing the chasm in the ice beneath, they act as pitfalls often wide and deep enough to swallow men and equipment. Safe trails through these areas are crooked and narrow and visual markers like flags and poles are obscured by poor visibility or destroyed by the elements.

The development of electronic techniques marks a giant stride forward in overcoming these hazards. According to Lynn A. Smitherman, ERDL project engineer, the electronic trail is now being extended over a hundred miles on the ice cap.

Work is continuing at the Fort Belvoir Laboratories and at General Mills, Inc., Minneapolis, Minn., to improve existing techniques and equipment. An improved design is already under test over an extended trail on the ice cap. A simplified one-wire trail-marking system which may reduce installation and maintenance costs is also under consideration.

**Royal Navy Divers at 176 Fathoms.**—A naval record was set recently in Norway's Sor Fjord by a British diver from H.M. salvage ship *Reclaim* with a descent of 1,060 ft. This was attained by Mr. G. O. Wookey, a commissioned bosun in the R.N., in a conventional observation chamber.

About seven other divers reached depths of about 965 ft.

A statement issued by *Reclaim* disclosed that visibility was aided by two lamps, one secured to the chamber and the other actually lowered to the bottom.

"During the dives, the occupants of the chamber were quite comfortable and could if necessary freshen air in the chamber from oxygen cylinders," the statement went on. "There was nothing much to see on the bottom except huge rocks and steep cliffs. The absence of fish life was noticeable, but all divers were impressed by the phosphorescence of the waters. Mr. Wookey carried out an endurance test of one hour at 1,060 ft."

In an emergency, the statement made clear, it would have been possible for the diver to release the slings and telephone cable and, by blowing the ballast tank clear of water, to give the chamber enough positive buoyancy to float to the surface and be towed back to the base ship.

**"Raincoat" for Military Items.**—A chemical "raincoat" in the form of a water-repellent will increase the service life of sandbags, tentage, knapsacks, tarpaulins, and other military items, according to tests conducted by the Corps of Engineers' Research and Development Laboratories, Fort Belvoir, Virginia.

Tests by the Laboratories showed that untreated bags, when used in areas that have high temperature and humidity, rot within three or four weeks. Fungicidal treatments will protect the sandbags for a year or more, when exposed to tropical conditions, but further service life can be achieved by the use of a water-repellent.

Use of the water-repellent to protect the fungicidal treatment not only increases the life of the fabrics, but also means a savings in fungicides. This is important inasmuch as many fungicides in use today contain copper and other ingredients that may be in short supply in event of a national emergency.

In six years of warfare, 1941 to 1945 and 1950 to 1952, the Armed Forces bought approximately 567 million sandbags at an estimated cost of 113 million dollars. Increasing the durability of this one item alone can result in significant savings to the tax payer.

**Animation Stand.**—A new, low cost animation stand has been announced by S.O.S. Cinema Supply Corp., of New York. Known as the TEL-ANIMASTAND, it permits the production of cartoons, titles, and other special effects.

A need was felt for a reasonably priced, self-contained animation stand for television stations, producers, educational institutions, and small laboratories. This led to the design of the TEL-ANIMASTAND which embodies features of the photo enlarger, movie camera, micrometer and railroad roundhouse. Even the heaviest 16-mm. or 35-mm. camera may be mounted upon a movable, counterbalanced vertical carriage which photographs the art work. Standard components and interchangeable parts increase the flexibility of the TEL-ANIMASTAND. The Add-A-Unit or "building block" idea was adopted, so the basic animation stand can later

be equipped with whatever refinements are desired.

Optical effects such as pans, angles, zooms, or quick closeups are accomplished by raising or lowering the camera. The compound table can turn a full 360° circle as well as travel to the front, back or either side. All basic movements associated with animation stands now used by the Industry are incorporated.

**Small Reactors for Schools and Laboratories.**—Design of a miniaturized low cost atomic energy reactor small enough to fit into any school, industrial, or medical laboratory has been announced by Atomics International, a division of North American Aviation, Inc.

The compact "Laboratory Reactor" is contained in a tank only 8 ft. high and 8 ft. in diameter, and can be installed in any existing laboratory building without the addition of any special facilities.

Priced at $55,000, the Laboratory Reactor can be constructed, installed and ready-to-run at any location in less than six months.

Designed specifically for small laboratories, the reactor will permit a wide range of research and training in the nuclear field. Operating at a power level of only 5 watts, the reactor will provide sufficient radiation for important studies in the engineering, biological and medical fields.

The "core" of the reactor, where atomic fission takes place, is a one foot stainless steel sphere containing about four gallons of enriched uranyl sulfate in a water solution. This core, surrounded by a six-inch thick lead shield, is housed in an eight foot tank filled with ordinary water. A number of exposure facilities and beam tubes are provided to permit several experiments to be carried on while the reactor is operating.

Only one operator is required to run the reactor which is controlled by a single console panel.

"The availability of this safe simplified low cost reactor will open the door to widespread study in the development in the nuclear field," according to Dr. A. B. Martin, Director of Technical Sales for Atomics International. "The Laboratory Reactor is expected to do for the nuclear science field what the Model T did for the automobile industry," Dr. Martin said.